

THESIS

THE SIZE OF COMBUSTION TURBINE PLANTS AS
AFFECTED BY CERTAIN DESIGN PARAMETERS

Shelley Elmer Rule

THE SIZE OF COMBUSTION TURBINE
PLANTS AS AFFECTED BY CERTAIN

DESIGN PARAMETERS G52-1

by
T

Shelley Elmer Rule

Department of Mechanical Engineering





Department of Mechanical Engineering
Massachusetts Institute of Technology
Cambridge 39, Massachusetts
May 16, 1952

Professor J. P. DenHartog
Chairman, Departmental Committee on Graduate Students
Department of Mechanical Engineering
Massachusetts Institute of Technology
Cambridge 39, Massachusetts

Dear Professor DenHartog:

In accordance with the requirements for graduation, I herewith submit a thesis entitled "The Size of Combustion Turbine Plants as Affected by Certain Design Parameters".

Sincerely yours,

Shelley Elmer Rule

Department of Mechanical Engineering
Massachusetts Institute of Technology
Cambridge 39, Massachusetts
May 16, 1947

Professor I. V. Dandekar
Chairman, Department of Mechanical Engineering
Department of Mechanical Engineering
Massachusetts Institute of Technology
Cambridge 39, Massachusetts

Dear Professor Dandekar:

In accordance with the requirements for graduation, I herewith submit a thesis entitled "The Role of
Mechanical Design in the Development of Certain Types of
Machines".

Sincerely yours,

Shelly West Nile

THE SIZE OF COMBUSTION TURBINE PLANTS
AS AFFECTED BY CERTAIN DESIGN PARAMETERS

BY

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Lieutenant Commander, U.S.N.

B.S. in M.E., Georgia School of Technology
(1939)

SUBMITTED IN PARTIAL FULFILLMENT OF THE
REQUIREMENTS FOR THE DEGREE OF MASTER
OF SCIENCE IN MECHANICAL ENGINEERING

at the

MASSACHUSETTS INSTITUTE OF TECHNOLOGY
(1952)

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THE BUREAU OF CONSTRUCTION THROUGH PLANS
AS APPLICABLE TO CERTAIN OTHER PLANNING

BY

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A B S T R A C T

THE SIZE OF COMBUSTION PLANTS AS AFFECTED BY CERTAIN DESIGN PARAMETERS

by

Shelley Elmer Rule

Submitted for the degree of Master of Science in Mechanical Engineering, on 16 May 1952.

As a means toward determining the effect of various parameters on combustion turbine plant size and weight for a particular net power output, three have been chosen as independent variables in which the plant may be expressed. These three are: the flow coefficient, (C_x/U) ; the blade length ratio, (L/d) ; the mass rate of flow, w . The size and weight of compressor, turbine and combustion components of a plant are stated in terms of these three, and the effect of blade aspect ratio on weight and volume is also mentioned. A specific example is used to illustrate the fact that the shaft speed ratio between compressor and turbine is intimately related to the optimum values of flow coefficient and blade length ratio to be chosen, whereas the effect of pressure ratio on this choice is very slight.

Thesis Supervisor:

Warren M. Rohsenow

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Assistant Professor of
Mechanical Engineering

ABSTRACT

THE EFFECT OF COMBUSTION PLANTS ON AIRBORNE NOISE
LEVELS

1

Summary of Results

Submitted for the degree of Master of Science in Mechanical Engineering, on 15 May 1961.

As a means toward determining the effect of various parameters on combustion turbine plant size and weight for aviation and power output, there have been obtained as follows: results in which the plant may be expressed. These three are: the two constants, (C_1, C_2) ; the plant length ratio, (L_1/L_2) ; the mass ratio of flow, (M_1/M_2) . The size and weight of compressor, turbine and combustion components of a plant are related to those of these three, and the effect of plant output ratio on weight and volume is also considered. A specific example is used to illustrate the fact that the plant would have better performance and volume is determined if related to the optimum values of these constants and plant length ratio. It is shown, however, that the effect of plant size on this choice is very slight.

Author: M. J. Johnson

Thesis Supervisor:

Assistant Professor of
Mechanical Engineering

Title:

ACKNOWLEDGEMENTS

The foundation upon which the latter part of this thesis rests, and which has been condensed to form the entire first portion, is the development of L. W. Shallenberg, presented as a thesis in 1951. I wish to express appreciation to the author both for that work and for the encouragement and assistance subsequently extended.

I desire also to express appreciation for the assistance and direction of Professor Rohsenow, the supervisor of this thesis, whose ever-willing advice contributed in large measure to the result.

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thesis rests, and which has been obtained so far as

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I. INTRODUCTION

The design of combustion turbine plants for optimum economy of construction and operation on the one hand, and for optimum weight and size against a specified useful output on the other, has been accompanied by much study, both along empirical and analytical lines.

In order to treat the problem by the ordinary mathematics, create a picture which can be grasped, cite an example which appears concrete and yet hold to a treatment sufficiently general to be useful, a considerable number of simplifying assumptions must be made -- and yet a minimum of them.

In the design of a power plant for many purposes, the weight of the plant and the amount of space it occupies are of paramount importance, whereas in any case both of them bear a relation to the first cost of the plant and its accommodation. Factors which may be thought to have an obvious bearing on size and weight may diverge from that widely in their actual effect.

The treatment herewith, as a small part of an extensive program, has drawn generously on what has gone before in attempting to focus on the effect of three particular variables on the size of rotating machinery: one geometric, one kinematic, and one a scale factor. The examination of each of the first two is limited to a span of "good practice", with the implication that trends shown within that

1. INTRODUCTION

The design of combustion turbine plants for ship-
and power of generation and operation on the one hand,
and for engine weight and size against a specified useful
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both along empirical and analytical lines.

In order to treat the problem in the ordinary
manner, a factor which can be regarded, also
as a factor which is not, and yet held to a limit-
and which is generally to be useful, a considerable number
of simplifying assumptions must be made -- and yet a mini-
mum of them.

In the design of a power plant for any purpose,
the weight of the plant and the amount of space it occupies
are of paramount importance, whereas in any other part of
the plant a relation to the first cost of the plant and its
maintenance. Factors which may be brought to bear on
other bearing on other and which may diverge from that
which in their actual effect.

The present review, as a small part of an exten-
sive program, has been purposely so that two more before
in attempting to focus on the effect of these particular
variables on the size of rotating machinery: one geometric,
one kinematic, and one a scale factor. The examination of
each of the first two is limited to a sign of "good prac-
tice", with the implication that trends shown within that

span will continue sufficiently far beyond it to reach the region, from the standpoint of each variable individually, of physical absurdity. Hence, if it is felt that the delineation of these spans of good practice is unjustified, a later extension or contraction of them will not alter the general aspect of the conclusions.

Subsequent to the treatment of rotating machinery, a general discussion of combustion equipment leads to relations, empirical but rational, for the weight and volume of that part of the plant.

Despite the fact that numerical constants have been inserted wherever possible to facilitate illustration, the influence of individual factors such as pressure ratio and cascade geometry is more truly represented than is the quantitative result. Even when the expressions arrived at permit a more general application, the tacit physical plant is a stationary or heavy propulsion plant in the 1000-10,000 horsepower range, but one borrowing from the light weight features of other types.

Whereas the method developed is in principle applicable to a wider range of plant configurations, only the simple OBT thermodynamic cycle between pressure ratios of five and ten has been investigated. An appropriate extension is to the CICBTX cycle, shown schematically in Fig. 1, using pressure ratios up to about fifteen. This is suggested as a subject for future study.

When all conditions are taken into account the
region, from the standpoint of some various industrial
of physical quantity. Hence, it is not the
definition of these areas of good quality is unaltered,
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According to the treatment of various machines,
general application of mechanical treatment leads to the
form, weight and material, for the weight and volume of
that part of the plant.
The first one has that mechanical treatment has been
improved wherever possible to facilitate illustration, the
influence of individual factors such as material weight
and weight capacity is more truly represented than in the
quantitative results. From this the expression appears as
results a more natural explanation, the final physical plant
is a stationary or semi-stationary plant in the 100-10,000
however, large, and the resulting from the light weight
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However, the method described is in principle applied
to a wider range of plant configurations, only the
single one characteristic of the system is that of
this and has been investigated. The mechanical system
also is in the 100-10,000 range, which is usually in the 1,
being produced ranging up to about fifteen. This is suggested
as a subject for future study.

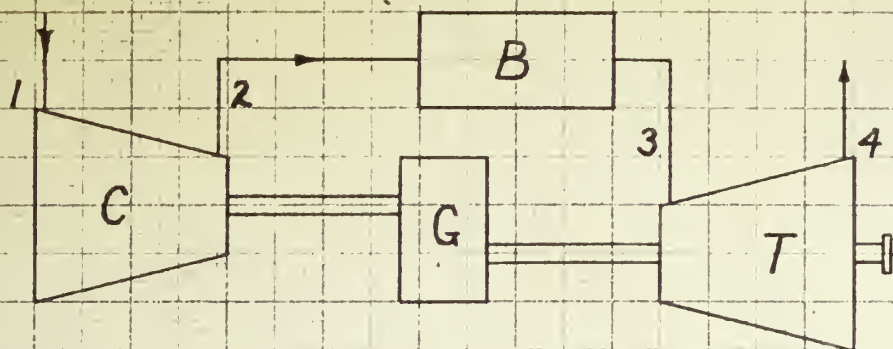


Fig. 1

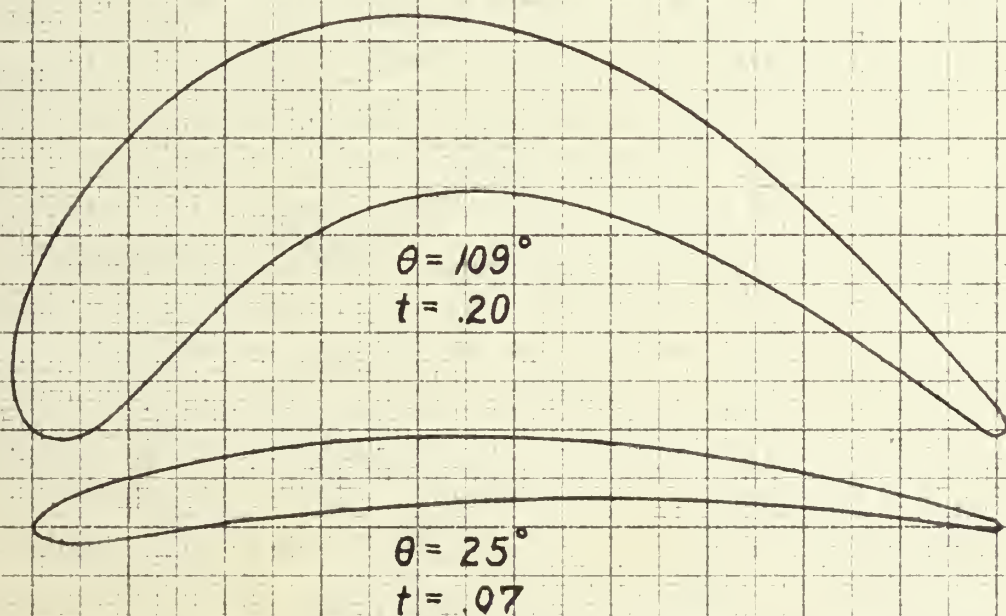
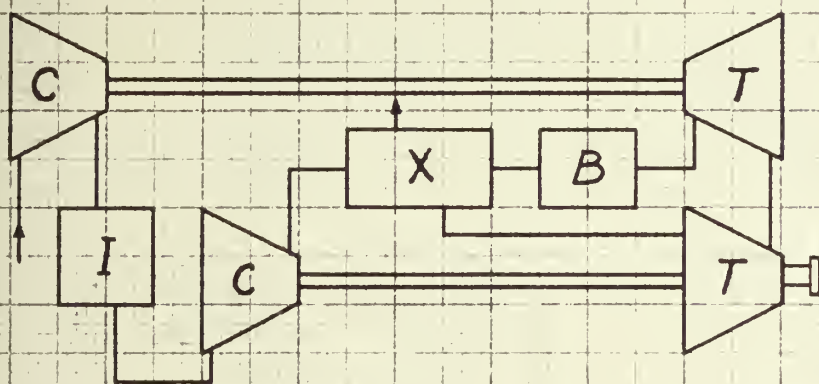


Fig. 3

II. BASIC RELATIONS AND DEFINITIONS

Assumptions involved in the analysis of plant components include: a constant pitch diameter throughout a particular machine, potential vortex flow, equal work done per turbine or compressor stage, a uniform axial velocity over the entire length of a machine. Compressor blading is assumed to be symmetrical in all stages except that the first and last are modified to meet substantially axial entry and discharge velocities. No pressure losses are considered in other than the turbine, and mechanical friction is overlooked. A mean value of specific heat is used in each of the ranges of compression, combustion and expansion, and the changes in mass flow and fluid properties due to fuel addition are neglected.

With these in mind, certain basic relations and some mathematical approximations which are used herein are outlined below.

Polytropic efficiencies:

$$\eta_c = \frac{r_c^\nu - 1}{(T_{03}/T_{01}) - 1} = \frac{r_c^\nu - 1}{r_c^\lambda - 1} = \frac{\eta_{sc}}{R_c^*}$$

$$T_{03} = r_c^\lambda T_{01} \quad (1)$$

$$\eta_t = \frac{1 - (T_{04}/T_{03})}{1 - r_t^{-\nu}} = \frac{1 - r_t^{-\mu}}{1 - r_t^{-\nu}} = R_t^* \eta_{st}$$

$$T_{04} = r_t^{-\mu} T_{03} \quad (2)$$

1.1. BASIC PRINCIPLES AND DEFINITIONS

Assumptions involved in the analysis of plant components are limited: a constant often diameter throughout a particular machine, rotational velocity flow, equal work done per turbine or compressor stage, a constant axial velocity over the entire length of a machine. Compressor blades are assumed to be symmetrical in all stages except that the first and last are modified to meet substantially axial entry and discharge velocities. No pressure losses are considered in other than the turbine, and mechanical friction is overlooked. A mean value of specific heat is used in each of the ranges of compression, combustion and expansion, and the changes in mass flow and fluid properties due to fuel addition are neglected.

With these in mind, certain basic relations and some mathematical approximations which are used herein are outlined below.

Polynomial approximations:

$$\begin{aligned}
 (1) \quad \frac{T_2}{T_1} &= \frac{1 - \frac{\gamma}{\gamma+1} \frac{V_1^2}{V_2^2}}{1 - \frac{\gamma}{\gamma+1} \frac{V_2^2}{V_1^2}} \\
 (2) \quad \frac{T_2}{T_1} &= \frac{1 - \frac{\gamma}{\gamma+1} \frac{V_1^2}{V_2^2}}{1 - \frac{\gamma}{\gamma+1} \frac{V_2^2}{V_1^2}}
 \end{aligned}$$

Stress parameter for tapered blades (ref 5):

$$\begin{aligned}\Sigma &\equiv \frac{\sigma_b}{T_{P_b}} = \frac{\omega^2 d^3}{2g} (L/d) \\ &= \frac{\omega^2 A}{2\pi g} = 4(U^2/2g)(L/d)\end{aligned}\quad (3)$$

where σ_b is due to rotation only.

This stress parameter commonly runs, for stationary or heavy propulsion plants, at about:

$$\begin{aligned}\Sigma_c &= 7000 \quad \text{ft.} \\ \Sigma_t &= 9000 \quad \text{ft.}\end{aligned}$$

while in portable and aircraft plants the figure

$$\Sigma_t = 14,000 \quad \text{ft.}$$

is frequently reached.

An examination of the mechanical properties of presently available alloys reveals that at the moderate temperature level of compressors the operating stresses are limited to about 36,000 to 42,000 psi by yield strength and endurance limits. At the temperature level of turbines the operating stresses are limited by creep rate to a value roughly proportional to T_{03}^{-4} .

Approximation of the relation between static and stagnation densities:

$$\frac{wRT}{P} = \frac{w}{\rho} = \frac{w}{\rho_0} \cdot \frac{\rho_0}{\rho}$$

$$\frac{\rho_0}{\rho} = \left[1 + \frac{k-1}{2} M^2\right]^{\frac{1}{k-1}} \approx \left[1 + \frac{1}{2} M^2\right]$$

Stationary for tapered blades (see 2):

$$\sum \frac{d^2}{r^3} = \frac{\omega^2 d^2}{r^3} = \frac{\omega^2 d^2}{r^3} (W/d)$$

(3)

$$\frac{\omega^2 d^2}{r^3} = \frac{\omega^2 d^2}{r^3} (W/d)$$

where d is the rotation only.

This stress is not a function of the rotation or

heavy production plants, at about:

$$\sum \frac{d^2}{r^3} = 7000 \quad \text{ft.}$$

$$\sum \frac{d^2}{r^3} = 9000 \quad \text{ft.}$$

Walls in concrete and masonry plants are thicker

$$\sum \frac{d^2}{r^3} = 14,000 \quad \text{ft.}$$

is frequently reached.

An examination of the mechanical properties of

presently available alloys reveals that of the materials

temperature level of the material the operating stresses

are limited to about 60,000 to 80,000 psi by yield strength

and endurance limits. At the temperature level of turbines

the operating stresses are limited to about 40,000 psi.

Values normally proportional to $\frac{1}{r^3}$.

Approximation of the relation between the two

stationary condition:

$$\frac{W}{r^3} = \frac{W}{r^3} = \frac{W}{r^3}$$

$$\left[\frac{1}{1 + \frac{1}{2} M^2} \right] \approx \left[\frac{1}{1 + \frac{1}{2} M^2} \right] = \frac{1}{2}$$

$$M^2 = \frac{C^2}{kgRT} \approx \frac{C_x^2}{kgRT_0}$$

By the continuity relation:

$$\frac{WRT}{P} = AC_x \approx \frac{WRT_0}{P_0} \left[1 + \frac{C_x^2/2g}{kRT_0} \right]$$

$$F_{M1} \equiv \frac{C_x^2/2g}{kRT_{01}} \quad ; \quad F_{M3} \equiv \frac{C_x^2/2g}{kRT_{03}} \quad (4)$$

For the compressor discharge (or combustor inlet):

$$AC_{x2} = \frac{WRT_{02}}{P_{02}} \left[1 + F_{M2} \right] = \frac{WRT_{01}r_c^\lambda}{P_{01}r_c} \left[1 + r_c^{-\lambda} F_{M1} \right]$$

$$= \frac{WRT_{01}}{P_{01}r_c} r_c + F_{M1} \quad (5)$$

A corresponding relation holds for the flow at turbine outlet, expressed in terms of T_{03} .

Mean density of fluid in the compressor:

$$\frac{\rho_1}{\rho_m} = \frac{\rho_1}{1/2(\rho_1 + \rho_2)} = \frac{2}{1 + (\rho_2/\rho_1)} \approx \frac{2}{1 + (\rho_{02}/\rho_{01})}$$

$$= \frac{2}{1 + (P_{02}/P_{01})(T_{01}/T_{02})}$$

$$= \frac{2}{1 + r_c} \equiv F_c \quad (6)$$

and correspondingly for the turbine:

$$\frac{p_4}{p_m} \approx \frac{2}{1+r_t} \frac{1}{1-\mu} \equiv F_t \quad (6a)$$

These functions are obtainable from Fig. 2.

Rotational speed of a machine, optimized from the standpoint that the parts under radial stress are all at the allowable limit expressed by the stress parameter Σ , may be determined by combining (3) and (4) with the geometrical relations of the wheel to yield

$$\text{RPM} = \frac{60\omega}{2\pi} = 30 \left[\frac{P(C_X/U)}{\pi w R T} \right]^{1/2} \left[\frac{2g^3 \Sigma^3}{(L/d)} \right]$$

Since the longest blades and consequently the highest blade stresses are found at compressor inlet and turbine outlet, relating the general equation to these two stations gives, for the compressor:

$$\text{RPM}_c = 272 \left[\frac{P_{01}(C_X/U)_1}{w R T_{01}(F_{m1}+1)} \right]^{1/2} \left[\frac{\Sigma_c^3}{(L/d)_1} \right]^{1/4} \quad (7)$$

and for the turbine:

$$\text{RPM}_t = 272 \left[\frac{P_{04}(C_X/U)_4}{w R T_{04}(F_{m4}+r_t^{-\mu})} \right]^{1/2} \left[\frac{\Sigma_t^3}{(L/d)_4} \right]^{1/4} \quad (7a)$$

It will be noted, when considering possible plant cycles, that a wide change in pressure ratio causes at most a 10% change in optimum RPM directly, but rather affects it via the air rate or w/P^* ratio. Turbine inlet temperature affects permissible rotational speed slightly in a direct manner, but much more heavily via its bearing on the stress

$$\frac{v}{v_0} = \frac{1}{1 + \frac{v}{v_0}}$$

These relations are obtainable from Eqs. 11.

Rotational speed of a machine, obtained from the

assumption that the points under radial stress are all at the allowable limit expressed by the known parameter Σ , may be determined by combining (3) and (4) with the geometrical relations of the wheel to field

$$\left[\frac{\Sigma}{(1/\lambda)^2} \right]^{1/2} \left[\frac{v_0}{v} \right]^{1/2} = \left[\frac{v_0}{v} \right]^{1/2} \left[\frac{v_0}{v} \right]^{1/2}$$

since the largest blades and consequently the highest blade stresses are found at compressor inlet and turbine exit, relating the general equation to these two stations gives, for the compressor:

$$\left[\frac{\Sigma}{(1/\lambda)^2} \right]^{1/2} \left[\frac{v_0}{v} \right]^{1/2} = \left[\frac{v_0}{v} \right]^{1/2} \left[\frac{v_0}{v} \right]^{1/2} \quad (7)$$

and for the turbine:

$$\left[\frac{\Sigma}{(1/\lambda)^2} \right]^{1/2} \left[\frac{v_0}{v} \right]^{1/2} = \left[\frac{v_0}{v} \right]^{1/2} \left[\frac{v_0}{v} \right]^{1/2} \quad (8)$$

It will be noted, when considering possible plant cycles, that a wide change in pressure ratio causes at most a 10% change in turbine exit velocity, but rather affects it the the air flow or W.P. ratio. Turbine inlet temperature affects practically rotational speed slightly in a direct manner, but much more heavily the gas bearing on the turbine

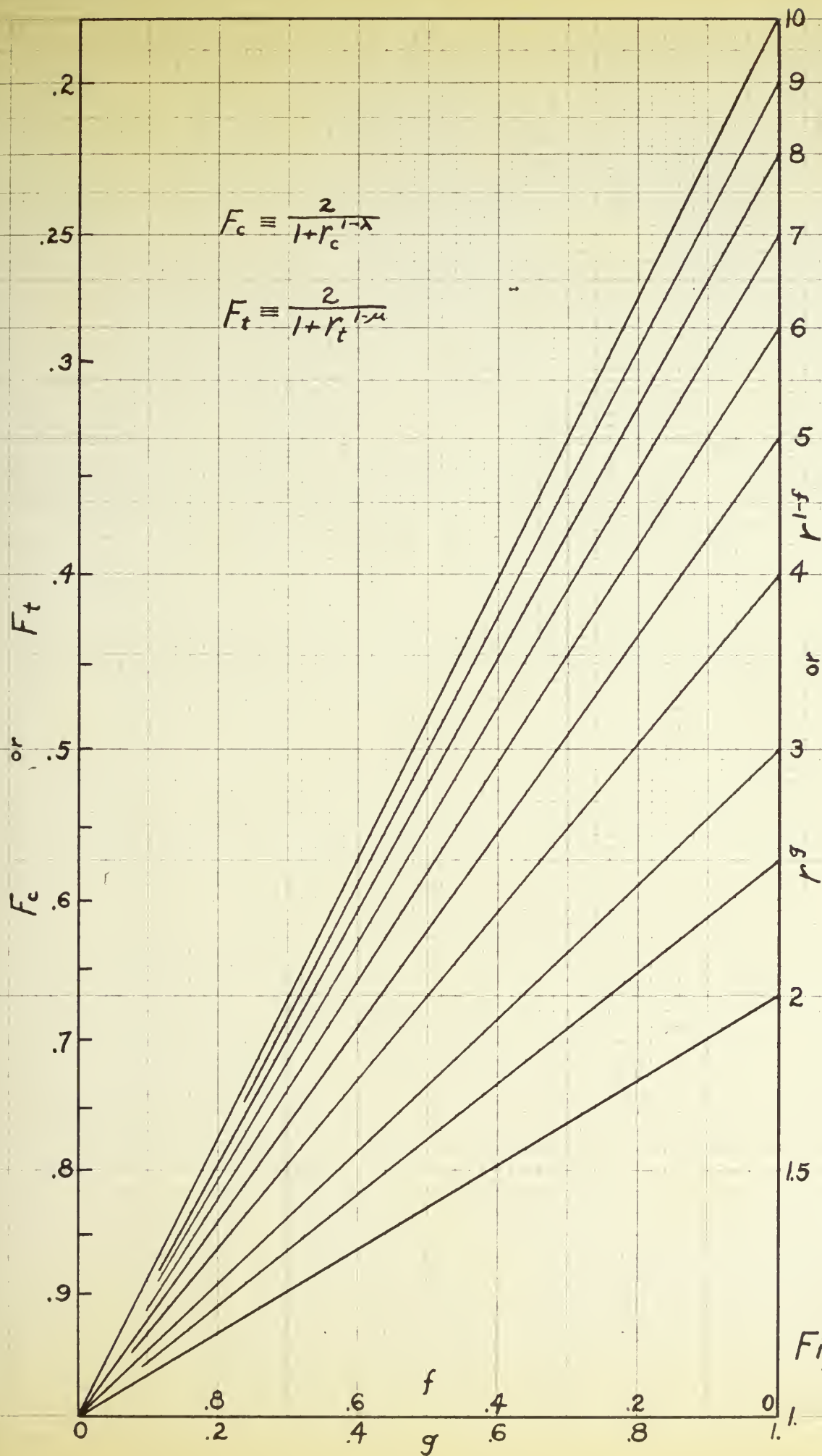


Fig. 2

parameter.

Pitch diameter of a rotor, optimized from the same standpoint as the RPM, is likewise determined to appear as:

$$d = \left[\frac{2wRT}{P(C_X/U)} \right]^{1/2} \left[2g \sum (L/d) \right]^{-1/4}$$

Relating this to compressor inlet and turbine outlet as before yields, for the compressor:

$$d_c = .282 \left[\frac{wRT_{01}}{P_{01}(C_X/U)_1} (F_{m1}+1) \right]^{1/2} \left[\sum_o (L/d)_1 \right]^{-1/4} \quad (8)$$

$$D_c = [1+(L/d)_1] d_c \quad (9)$$

and for the turbine:

$$d_t = .282 \left[\frac{wRT_{02}}{P_{02}(C_X/U)_2} (F_{m2}+r_t^{-1}) \right]^{1/2} \left[\sum_t (L/d)_2 \right]^{-1/4} \quad (8a)$$

$$D_t = [1+(L/d)_2] d_t \quad (9a)$$

The same remarks regarding the effects of pressure ratio and turbine inlet temperature on the RPM apply generally to the diameter relations above, but of course in the reverse direction.

Power requirement and output, assuming a mean value of C_p and of k to be applicable over the temperature spans of each component, may be expressed, with (1) and (2), as:

$$P^* = w(h_1-h_2) \approx wC_p(T_{01}-T_{02})$$

which becomes, for the compressor:

PARAMETER.

Piston diameter of a motor, optimized from the same stand-
point as the RIM, is likewise determined to appear as:

$$D = \left[\frac{2 \pi \tau}{\pi (D \sqrt{V})} \right]^{1/2} \left[\frac{2 \pi \tau}{\pi (D \sqrt{V})} \right]^{-1/2}$$

Relating this to compressor inlet and turbine outlet as

before yields, for the compressor:

$$D_c = \left[\frac{2 \pi \tau}{\pi (D \sqrt{V})} \right]^{1/2} \left[\frac{2 \pi \tau}{\pi (D \sqrt{V})} \right]^{-1/2} \quad (8)$$

$$D_c = \left[1 + (L \sqrt{V}) \right]^{1/2} \quad (9)$$

and for the turbine:

$$D_t = \left[\frac{2 \pi \tau}{\pi (D \sqrt{V})} \right]^{1/2} \left[\frac{2 \pi \tau}{\pi (D \sqrt{V})} \right]^{-1/2} \quad (10)$$

$$D_t = \left[1 + (L \sqrt{V}) \right]^{1/2} \quad (11)$$

The same formulae regarding the effects of pressure ratio
and turbine inlet temperature on the RIM apply generally
to the distiller relations above, but of course in the
reverse direction.

Power requirement and cooling, assuming a mean value of ϕ
and of k to be applicable over the temperature spans of each
component, may be expressed, with (1) and (2), as:

$$P = W(h_1 - h_2) \approx W_p (T_{01} - T_{02})$$

which becomes, for the compressor:

$$P_c^* = w C_{pc} T_{o_1} (r_c^{\gamma} - 1) \quad (10)$$

and for the turbine, with negligible leaving loss:

$$P_t^* = w C_{pt} T_{o_2} (1 - r_t^{\gamma}) \quad (10a)$$

Allowing for leaving losses but assuming the leaving velocity nearly axial, or with negligible whirl, the last becomes:

$$P_t^* = w \left[C_{pt} T_{o_2} (1 - r_t^{\gamma}) - \frac{C_x^2}{2gJ} \right] \quad (10b)$$

In a plant cycle the effect of pressure ratio on air rate, w/P^* , is well known, and the curve of Fig. 4 is representative for a plant in which the output takes the form of shaft power. Since, knowing the cycle temperatures and the probable component efficiencies, the air rate is a known function of pressure ratio only, it will frequently be convenient to arrange an expression for, say, component weight in the form:

$$W^* = F(\text{cycle conditions}) \cdot F(\text{pressure ratio})$$

(10)

$$T_0 = W_0 T_0 (1 - \gamma_0)$$

and for the turbine, with negligible leaving loss:

(10a)

$$T_0 = W_0 T_0 (1 - \gamma_0)$$

Allowing for leaving losses but assuming the leaving velocity

is nearly axial, or with negligible whirl, the loss

becomes:

(10b)

$$T_0 = W_0 T_0 (1 - \gamma_0) \left[1 - \frac{1}{2} \frac{U^2}{C^2} \right]$$

In a plant cycle the effect of pressure ratio on air rate, W/T_0 , is well known, and the curve of fig. 4 is representative for a plant in which the output takes the form of shaft

power. Since, knowing the cycle temperatures and the

probable component efficiencies, the air rate is a known

function of pressure ratio only, it will frequently be convenient to express an expression for, say, component weight

in the form:

$$W = f(\text{cycle conditions}) \cdot f(\text{pressure ratio})$$

Effect of Pressure Ratio on Component Size and Weight

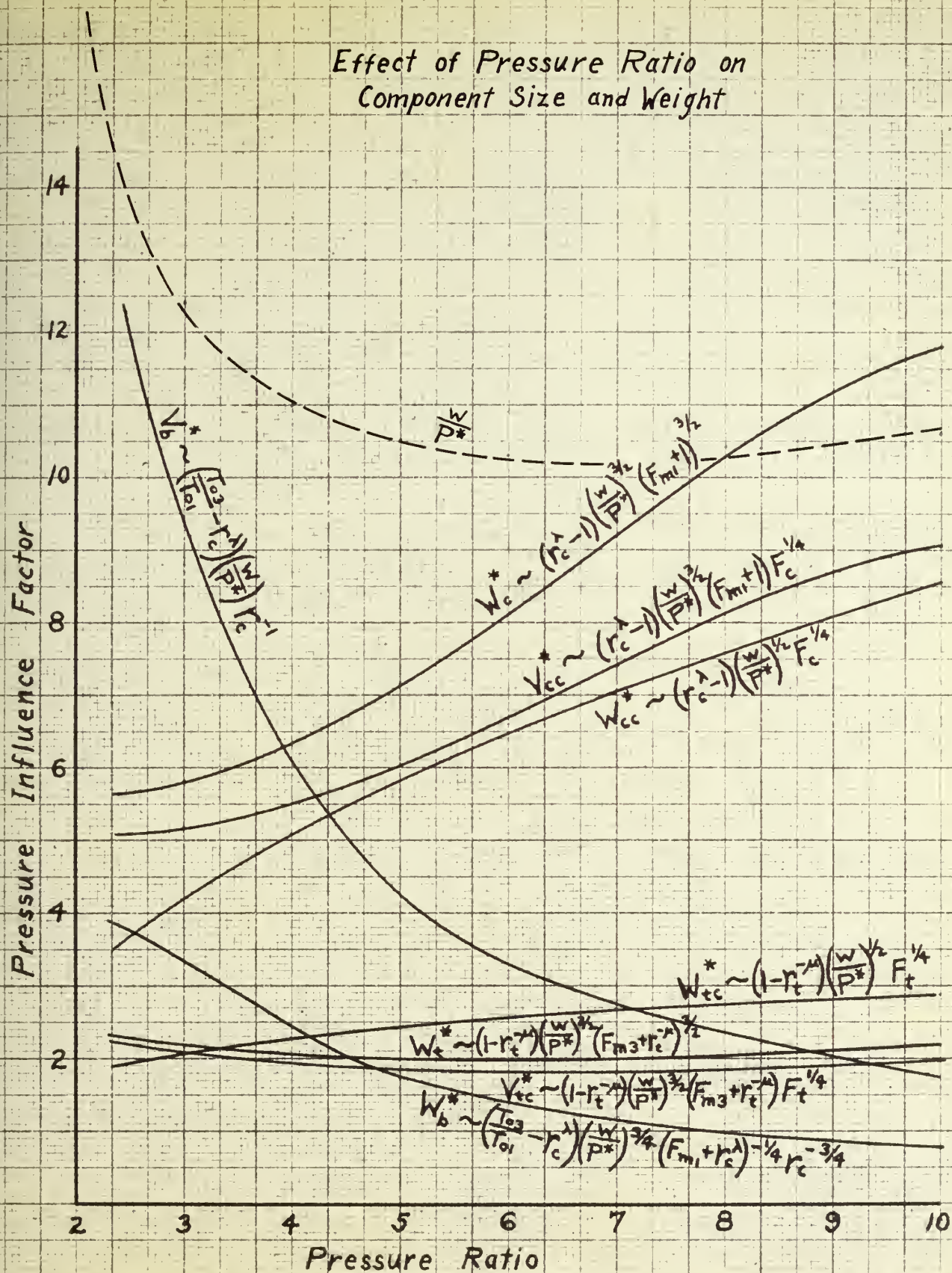


Fig. 4

III. MATCHING COMPONENTS

In matching components to form a plant cycle certain requirements are to be met, some of them necessary and obvious if the plant is to operate at all, others desirable and implicit if it is to give optimum satisfaction. For the simple cycle single-shaft jet propulsion plant (CBTJ), exemplified by aircraft installations for instance, the following hold:

$$\begin{aligned} \text{RPM}_c &= \text{RPM}_t & w_c &= w_t \\ r_c P_{o1} &= r_t P_{o2} & R_c &= R_t \\ C_{pc} &< C_{pb} < C_{pt} \end{aligned}$$

Taking the ratio of (8) to (8a) and using (3) to eliminate gives:

$$\frac{d_c}{d_t} = \left[\frac{r_c T_{o1} (F_{m1} + 1) (C_x/U)_c (L/d)_c}{r_t T_{o2} (F_{m2} + r_t^{-1}) (C_x/U)_t (L/d)_t} \right]^{1/3} \quad (11)$$

or:

$$\frac{d_c}{d_t} = \left[\frac{r_c T_{o1} (F_{m1} + 1) (C_x/U)_c \sigma_{bt}}{r_t T_{o2} (F_{m2} + r_t^{-1}) (C_x/U)_t \sigma_{bc}} \right] \quad (11a)$$

the latter holding only if \mathcal{T} is the same for both machines, and the blade material is such that ρ_b is the same for both. Since turbine and compressor power are substantially equal at all times (10) and (10a) combine to give:

$$C_{pc} T_{o1} (r_c^\lambda - 1) = C_{pt} T_{o2} (1 - r_t^{-1})$$

III. MATHEMATICAL CONSIDERATIONS

In selecting components to form a plant state certain requirements are to be met, some of them necessary and obvious if the plant is to operate at all, others desirable and implicit if it is to give optimum performance. For the simple cycle analysis which is the subject of this paper, the following hold:

$$\begin{aligned} P_{01} &= P_{02} \\ P_{01} &= P_{02} \\ P_{01} &= P_{02} \end{aligned}$$

Taking the ratio of (8) to (8a) and using (9) to

eliminate gives:

$$(11) \quad \frac{dP_{01}}{dP_{02}} = \frac{P_{01}^{T_{01}+1} (P_{02}^{T_{02}+1})}{P_{02}^{T_{01}+1} (P_{01}^{T_{02}+1})}$$

or:

$$(12) \quad \frac{dP_{01}}{dP_{02}} = \frac{P_{01}^{T_{01}+1} (P_{02}^{T_{02}+1})}{P_{02}^{T_{01}+1} (P_{01}^{T_{02}+1})}$$

the latter holding only if T is the same for both machines, and the blade material is such that P_{01} is the same for both. Since turbine and compressor power are substantially equal at all times (10) and (12a) combine to give:

$$P_{01}^{T_{01}+1} (P_{02}^{T_{02}+1}) = P_{02}^{T_{01}+1} (P_{01}^{T_{02}+1})$$

The useful power output of such a plant takes the form of thrust, dependent on jet velocity.

$$\frac{C_j}{2gJ} = C_{pb}(T_{o4} - T_j) = C_{pb}T_{o4} \left[1 - \frac{T_j}{T_{o4}} \right]$$

$$C_j = 2gJC_{pb}T_{o4} \left[r_t^{-1/\mu} - r_c^{-\nu} r_t^{\nu-1/\mu} \right]$$

or in terms of the compressor pressure ratio only:

$$C_j = 2gJC_{pb}T_{o4} \left[1 - \frac{C_{pc}T_{o1}}{C_{pt}T_{o4}} (r_c^\lambda - 1) \right] \left[1 - \left(1 - \frac{C_{pc}T_{o1}}{C_{pt}T_{o4}} (r_c^\lambda - 1) \right)^{-1/\mu} r_c^{-\nu} \right] \quad (12)$$

In the special case of static thrust:

$$\frac{F_s}{w} = \frac{C_j}{g}$$

For the simple cycle stationary or heavy propulsion plant (CBTG), exemplified by the railway locomotive unit, the following hold:

$$G \cdot \text{RPM}_c = \text{RPM}_t \quad \text{where } G \text{ is a gear ratio}$$

$$w_c = w_t$$

$$R_c = R_t$$

$$P_{o1} = P_{o4}$$

$$r_c = r_t$$

$$C_{pc} < C_{pb} < C_{pt}$$

Substituting (7) and (7a) into the first of these relations, and making use of the others where appropriate:

$$G^2 = \frac{T_{o1}(F_{M1}+1)(C_X/U)_4}{T_{o4}(F_{M4}+r^{-1/\mu})(C_X/U)_1} \left[\frac{(L/d)_1 \sum_t^2}{(L/d)_4 \sum_c^3} \right]^{1/2} \quad (14)$$

or, using (3) to eliminate \sum leaves:

The useful power output of such a plant is given by the formula
 of energy, dependent on its velocity.

$$\left[\frac{1}{2} - \frac{1}{2} \left(1 - \frac{v^2}{c^2} \right) \right] \left[1 - \frac{v^2}{c^2} \right]$$

$$\left[1 - \frac{v^2}{c^2} \right] \left[1 - \frac{v^2}{c^2} \right]$$

or in terms of the relativistic momentum with which

$$\left[1 - \frac{v^2}{c^2} \right] \left[1 - \frac{v^2}{c^2} \right] \left[1 - \frac{v^2}{c^2} \right] \left[1 - \frac{v^2}{c^2} \right]$$

In the special case of a static system:

$$\frac{1}{2} - \frac{1}{2} \frac{v^2}{c^2}$$

For the simple case of a stationary or nearly stationary
 plant (OBS), described by the relativistic energy,
 the following holds:

$$E_{rel} = \frac{1}{2} m_0 c^2 \left(1 + \frac{v^2}{c^2} \right)$$

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Substituting (7) and (8) into the first of these
 relations, and making use of the other two, we obtain:

$$\left[1 - \frac{v^2}{c^2} \right] \left[1 - \frac{v^2}{c^2} \right] \left[1 - \frac{v^2}{c^2} \right] \left[1 - \frac{v^2}{c^2} \right]$$

or, using (5) to eliminate Σ leaves:

$$G = \frac{T_{02}(F_{m2} + r^{-\lambda})(C_X/U)_1(L/d)_1 d_c^3}{T_{01}(F_{m1} + 1)(C_X/U)_2(L/d)_2 d_t^3} \quad (15)$$

The net power output of this type of plant is:

$$p_n^* = p_t^* - p_c^*$$

By (10) and (10a) then:

$$\frac{p^*}{w} = J C_{pc} T_{01} \left[\frac{C_{pt} T_{02} (1 - r^{-\lambda}) - (r^{\lambda} - 1)}{C_{pc} T_{01}} \right] \quad (16)$$

And the cycle thermal efficiency becomes:

$$\eta = \frac{C_{pt}(T_{02}/T_{01})(1 - r^{-\lambda}) - C_{pc}(r^{\lambda} - 1)}{C_{pb}[(T_{02}/T_{01}) - r^{\lambda}]} \quad (17)$$

$$(15) \quad \frac{\sum_{i=1}^n \delta_i(b \setminus L)_i (b \setminus L)_i^{\frac{1}{2}} (c \setminus L)_i^{\frac{1}{2}}}{\sum_{i=1}^n \delta_i(b \setminus L)_i (b \setminus L)_i^{\frac{1}{2}} (c \setminus L)_i^{\frac{1}{2}} + 1} = 0$$

The last part of this type of class is:

$$\frac{1}{n} \sum_{i=1}^n \delta_i(b \setminus L)_i^{\frac{1}{2}} (c \setminus L)_i^{\frac{1}{2}} = 0$$

By (16) and (17) then:

$$(18) \quad \left[\frac{\sum_{i=1}^n \delta_i(b \setminus L)_i^{\frac{1}{2}} (c \setminus L)_i^{\frac{1}{2}}}{\sum_{i=1}^n \delta_i(b \setminus L)_i^{\frac{1}{2}} (c \setminus L)_i^{\frac{1}{2}} + 1} \right] \frac{1}{n} \sum_{i=1}^n \delta_i(b \setminus L)_i^{\frac{1}{2}} (c \setminus L)_i^{\frac{1}{2}} = 0$$

and the same result is obtained by:

$$(19) \quad \frac{\sum_{i=1}^n \delta_i(b \setminus L)_i^{\frac{1}{2}} (c \setminus L)_i^{\frac{1}{2}}}{\sum_{i=1}^n \delta_i(b \setminus L)_i^{\frac{1}{2}} (c \setminus L)_i^{\frac{1}{2}} + 1} = 0$$

IV. DETERMINATION OF THE STAGE

Thus far, no consideration has been given to aerodynamic relations or the factors affecting efficiency within a stage, nor with stage dimensions. There are to be developed an expression for the axial width of the blade row, the blade spacing, and the number of stages required. The length and volume of machine rotors is readily obtained then from these expressions and those previously set forth.

Zweifel (ref. 1) has developed an aerodynamic load coefficient

$$\Psi_t = 2 \sin^2 \beta_2 (\cot \beta_2 - \cot \beta_1) \frac{1}{\delta}$$

based upon the attainable pressure distribution around an airfoil, which coefficient he shows to have a value very near eight-tenths for minimum pressure loss and minimum drag/lift ratio in a cascade. This significantly corresponds to the preferred design deflection angle of eight-tenths that for which stall occurs, presented by Howell (ref. 2). The work done in a stage may be written as:

$$\frac{W}{U^2/2g} = 2(C_x/U)(\cot \beta_2 - \cot \beta_1)$$

Combining the above two expressions, the optimum solidity of a blade row may be stated as:

$$\delta = 1.25 \frac{\sin^2 \beta_2}{(C_x/U)} \frac{W}{U^2/2g} \quad (18)$$

On the other hand, Schnittger (ref. 3) has indicated that for optimum stage efficiency the camber of a blade is related

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$$\psi = \frac{1}{2} \left(\frac{V}{V_0} \right)^2 \left(\cos^2 \alpha - \cos^2 \beta \right)$$

Based upon the attainable pressure distribution around an

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The work done in a stage may be written as:

$$\frac{W}{U^2 \sqrt{S}} = 2 \left(\frac{V}{U} \right) (\cos^2 \alpha - \cos^2 \beta)$$

Combining the above two expressions, the optimum solidity

of a blade row may be stated as:

$$(12) \quad \frac{W}{U^2 \sqrt{S}} = 1.62 \frac{V}{U} \frac{2 \sqrt{S}}{U} \left(\frac{V}{U} \right)$$

On the other hand, Swirell (ref. 1) has indicated that

for optimum stage efficiency the number of a blade is related

to the gas leaving angle by:

$$.5 < \theta/\beta_2 < .55$$

and further that the fluid deflection angle in decelerating cascades is related to this leaving angle by:

$$\epsilon = .307\beta_2(c/s)^{1/2} \approx .307\beta_2\delta^{1/2}$$

Hence, by combination it follows that:

$$\beta_1 = \beta_2 (1 - .307\delta^{1/2})$$

and by substitution in Zweifel's relation above, the optimum solidity becomes:

$$\delta = 2.5 \sin^2 \beta_2 \left[\cot \beta_2 - \cot(1 - .307\delta^{1/2})\beta_2 \right] \quad (19)$$

which is readily solved by trial, on the first attempt estimating $\delta = 1$ on the right.

Figures 5 and 6 illustrate the nomenclature used above. The latter is a reproduction of the curves presented by Zweifel on which has been drawn the line of maximum turbine stage efficiency given by Hawthorne (ref. 4). The difference in notation between these figures and that used herein should be noted.

Gas bending stress in a blade may be found from the fundamental formula of mechanics:

$$\sigma_g = \frac{my}{I} = \frac{m}{I/y}$$

wherein the moment is:

$$\begin{aligned} m &= 1/2 LF = 1/2 d(L/d) \frac{WU^2}{2g} \\ &= \frac{W}{2} (L/d) \frac{W}{U^2/2g} (U^2/2g) \end{aligned}$$

to the vanishing angle by:

$$\alpha > \alpha_0 > \alpha_1$$

and further that the final deflection angle is determined

as follows in relation to the limiting angle by:

$$\alpha = 2.307 \alpha_0 \approx 2.307 \alpha_1$$

where, by comparison it follows that:

$$\alpha_1 = \alpha_0 (1 - 0.004)$$

and by substitution in Swell's relation above, the optimum

relation becomes:

$$\alpha = 2.307 \alpha_0 \left[\cos^2 \alpha_0 - \cos^2 (1 - 0.004) \alpha_0 \right]^{1/2} \quad (2)$$

which is readily solved by trial, on the first attempt esti-

mating $\alpha = 1$ on the right.

Figures 5 and 6 illustrate the relationships noted above.

The latter is a reproduction of the curves presented by

Swell on which has been drawn the line of maximum deflection

angle already given by Newton's (ref. 2). The differences

in notation between these figures and that used herein should

be noted.

The bending stress in a blade may be found from the

fundamental formula of mechanics:

$$\sigma = \frac{M}{I} y$$

wherein the terms are:

$$\sigma = \text{stress} = \frac{W}{A} \quad I = \text{moment of inertia} = \frac{W}{12} \quad y = \text{distance from neutral axis}$$

$$\sigma = \frac{W}{A} \left(\frac{L}{2} \right) \left(\frac{1}{2} \right) \left(\frac{1}{2} \right)$$

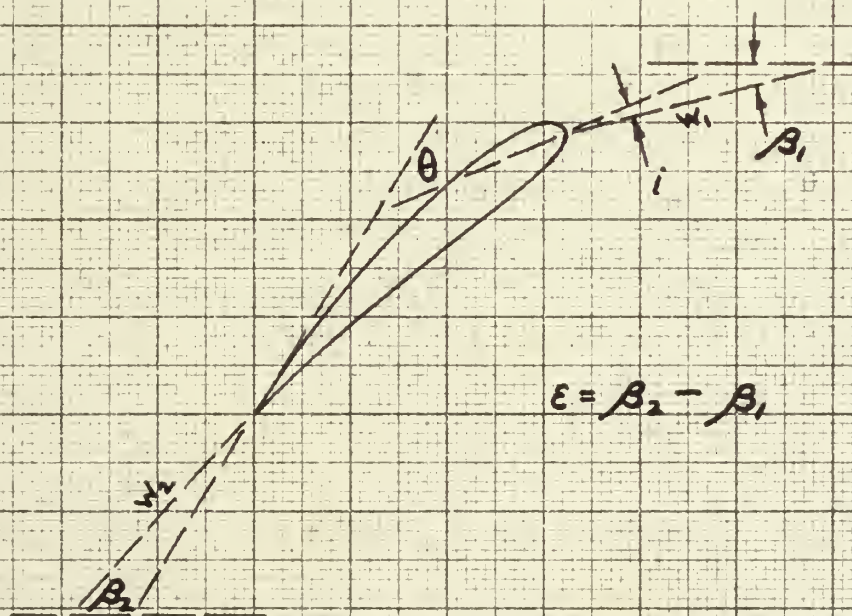
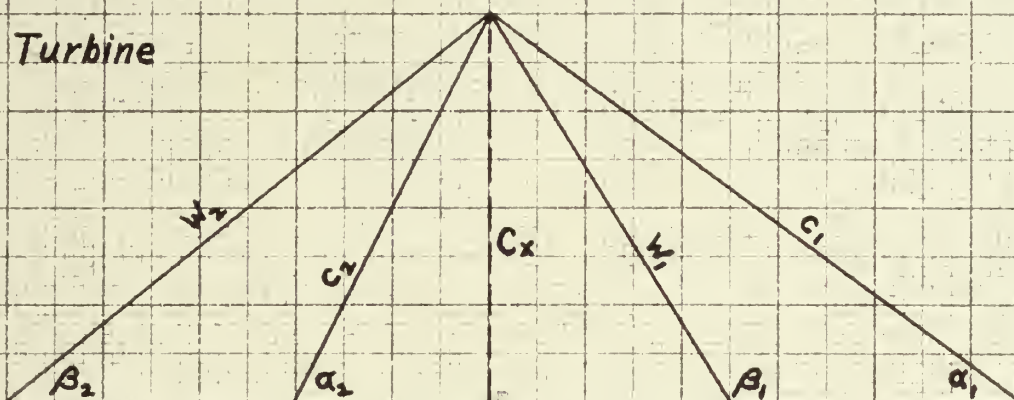
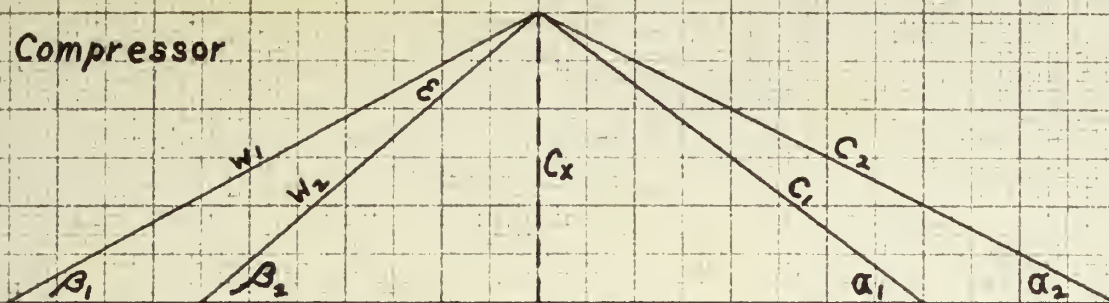
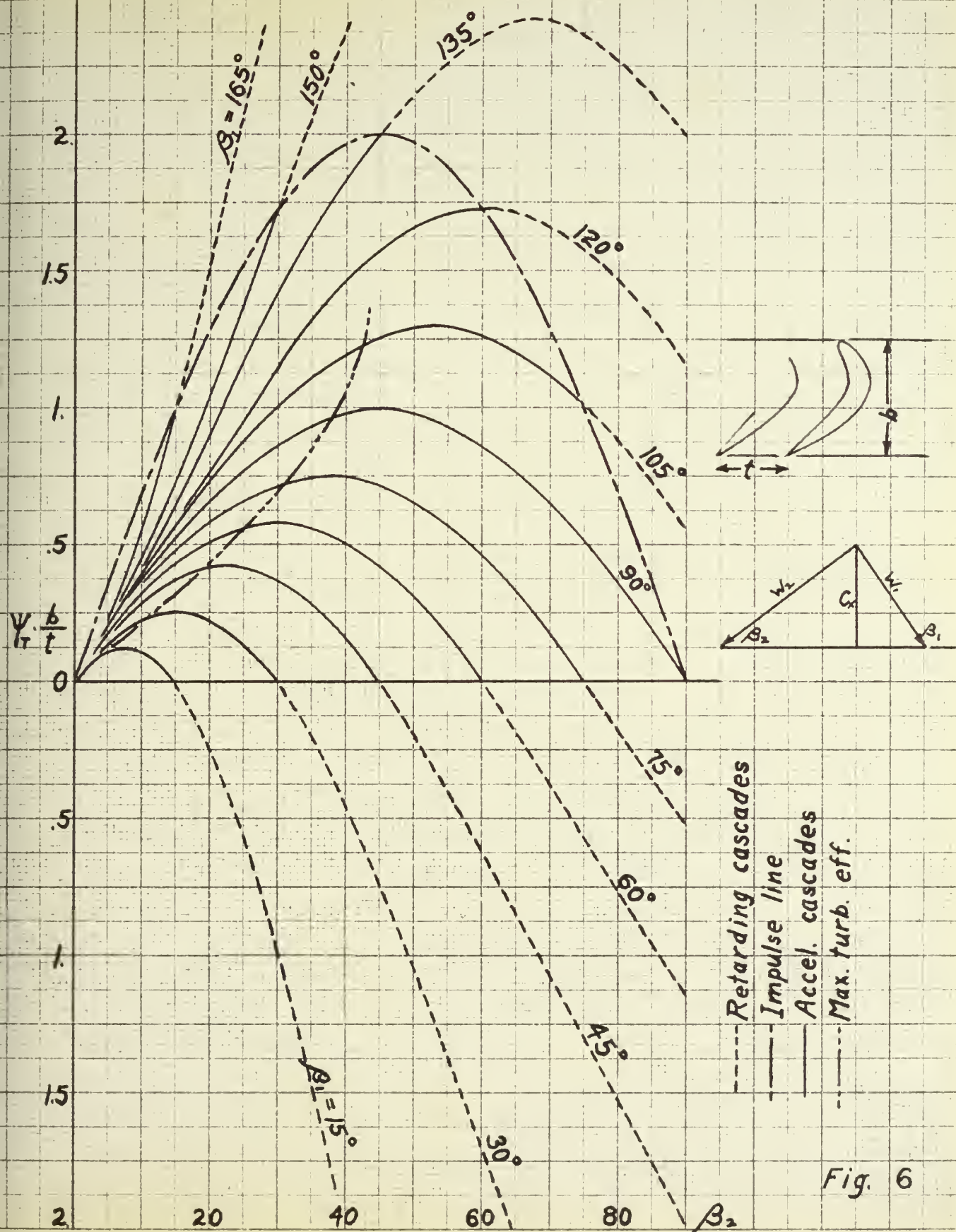


Fig. 5

Optimum Blade Spacing - Zweifel



But from the geometry, and (18) above:

$$Z = \frac{\pi \delta}{(L/d)} = \frac{1.25 \pi \delta \sin^2 \beta_s}{(L/d)(C_x/U)} \frac{W}{U^3/2g}$$

Hence, by substitution:

$$n = \frac{w(L/d)^3 (C_x/U)(U^3/2g)}{1.25\pi \omega \delta \sin^2 \beta_s}$$

In order to evaluate the blade section modulus, I/y , various airfoil section plans were measured to determine the effect of camber and thickness ratio in the moment of inertia and extreme fiber distance. A standard thickness distribution was used, with camber varied 25° to 109° , thickness ratio varied 7% to 10%, based on a parabolic camber line. The extreme of the shapes so measured are shown in Fig. 3. From this there was deduced (θ in radians here):

$$\frac{I}{y} = \frac{b^3}{1000} \left[.35\theta^{2.4} - 85t^2 \right] \quad (20)$$

Substituting this in the above:

$$\sigma_g = \frac{2280 w (L/d)^3 (C_x/U)(U^3/2g)}{\pi \omega \delta b^3 \sin^2 \beta_s F_b} \quad (21)$$

wherein

$$F_b \equiv \theta^{2.4} - 242.5t^2$$

Replacing the aspect ratio by its definition:

$$\delta \equiv (L/b) = (L/d)d/b$$

and using (3) to eliminate ω , d and $(U^3/2g)$:

$$b^3 = \frac{2280}{4\pi\sqrt{2g}} \frac{w(L/d)^{1/2} (C_x/U)^{1/2}}{\sigma_g F_b \sin^2 \beta_s}$$

and from the geometry, and (12) above:

$$\frac{1}{\sin \delta} = \frac{w \delta \sin \delta}{(1/\delta)(1/\delta)(1/\delta)} = \frac{w \delta \sin \delta}{1/\delta^3}$$

hence, by substitution:

$$\frac{1}{\sin \delta} = \frac{w \delta \sin \delta (1/\delta)(1/\delta)(1/\delta)}{1/\delta^3}$$

In order to evaluate the plate section modulus, I_p , various initial section planes were measured to determine the effect of center and thickness ratio in the moment of inertia and extreme fiber distance. A standard thickness distribution was used, with center varied 25° to 100°, thickness ratio varied 75 to 100, based on a parabolic center line. The extreme of the shape so measured are shown in Fig. 3. From this figure was deduced (θ in radians here):

$$(50) \quad \frac{1}{\sin \delta} = \frac{p}{1000} \left[\frac{1}{\sin \delta} - \frac{1}{\sin \delta} \right]$$

Substituting this in the above:

$$(51) \quad \frac{1}{\sin \delta} = \frac{p \delta \sin \delta (1/\delta)(1/\delta)(1/\delta)}{1/\delta^3}$$

where:

$$I_p = \frac{1}{12} \pi \delta^3 \sin^2 \delta$$

Repeating the above ratio by its definition:

$$\delta = (1/p) = (1/\delta)(1/\delta)$$

and using (5) to eliminate ω , δ and $(1/\delta)$:

$$\frac{1}{\sin \delta} = \frac{p \delta \sin \delta (1/\delta)(1/\delta)(1/\delta)}{1/\delta^3}$$

AIRFOIL SECTION MODULI

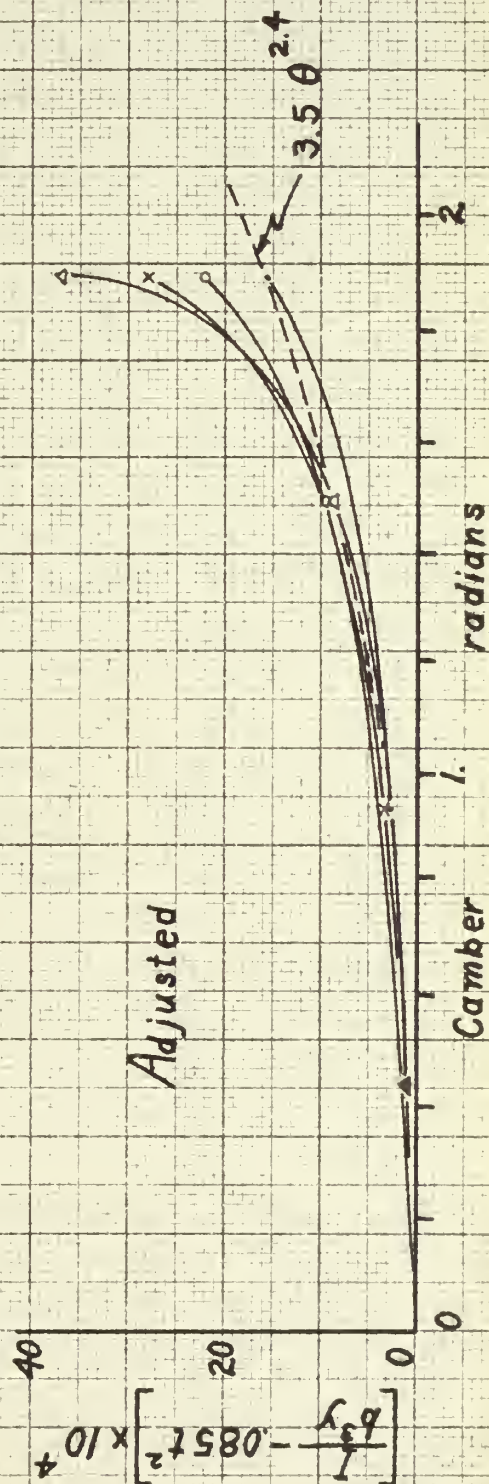
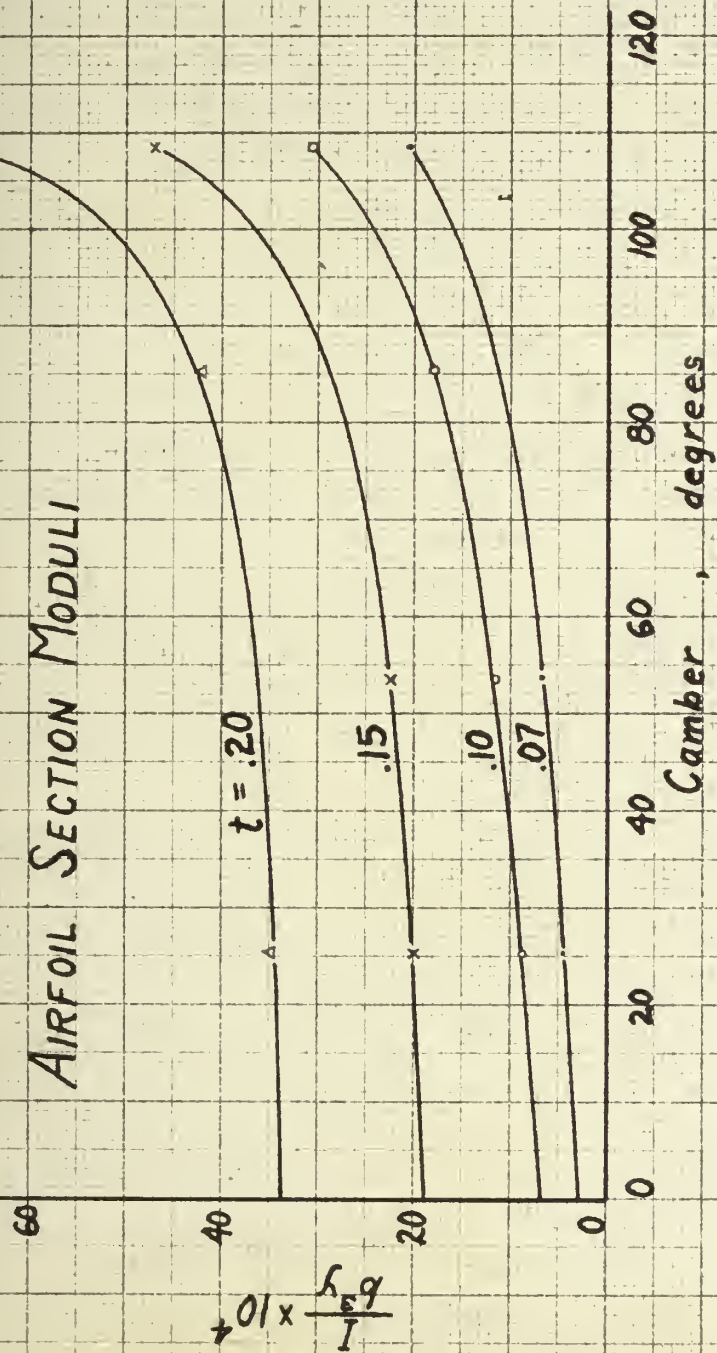


Fig. 7

wherefrom:

$$b = \frac{4.75}{\sin \beta_s} \left[\frac{w(C_x/U)}{\sigma_g F_b} \right]^{1/2} \left[(L/d) \Sigma \right]^{1/4}$$

Since in obtaining the overall rotor length the mean stage width is desired, a mean value of (L/d) must be obtained for use in the foregoing, and (L/d) in turn is proportional to local density if C_x is to be maintained constant. Hence, by (8), for the compressor:

$$(L/d)_{mc} = (L/d)_1 (\rho_1/\rho_m) = (L/d)_1 F_t \quad (22)$$

and by (6a), for the turbine:

$$(L/d)_{mt} = (L/d)_s F_t \quad (22a)$$

These then lead to the compressor row width:

$$b_{mc} = \frac{4.75}{\sin \beta_s} \left[\frac{w(C_x/U)_1}{\sigma_g F_{bc}} \right]^{1/2} \left[(L/d)_1 F_c \Sigma_c \right]^{1/4} \quad (23)$$

and the turbine's:

$$b_{mt} = \frac{4.75}{\sin \beta_s} \left[\frac{w(C_x/U)}{\sigma_g F_{bt}} \right]^{1/2} \left[(L/d)_s F_t \Sigma_t \right]^{1/4} \quad (23a)$$

The number of stages required depends jointly on the work to be done in a machine and the amount of work efficiently attainable per stage, or:

$$N = \frac{\Delta h}{W_s \eta_{sc}}$$

This becomes, for the compressor:

$$p = \frac{4.75}{\sin 35} \left[\frac{w(\sqrt{1/2})}{\sigma_{\sqrt{1/2}}} \right] \left[\frac{\sqrt{1/2}}{\sqrt{1/2}} \right]$$

Since in obtaining the overall total length the mean

value which is desired, a mean value of (L/A) may be of-

tain for use in the foregoing, and (L/A) in turn is propor-

tional to local density if σ is to be maintained constant.

Hence, by (8), for the compressor:

$$(L/A)_{\text{comp}} = (L/A)_1 (v_1/v_2) = (L/A)_1 v_1$$

and by (9), for the turbine:

$$(L/A)_{\text{turb}} = (L/A)_2 v_2$$

These then lead to the compressor flow width:

$$(93) \quad p_{\text{comp}} = \frac{4.75}{\sin 35} \left[\frac{w(\sqrt{1/2})}{\sigma_{\sqrt{1/2}}} \right] \left[\frac{\sqrt{1/2}}{\sqrt{1/2}} \right]$$

and the turbine's:

$$(94) \quad p_{\text{turb}} = \frac{4.75}{\sin 35} \left[\frac{w(\sqrt{1/2})}{\sigma_{\sqrt{1/2}}} \right] \left[\frac{\sqrt{1/2}}{\sqrt{1/2}} \right]$$

The number of stages required depends jointly on the work to

be done in a machine and the amount of work efficiently

obtainable per stage, or:

$$N = \frac{\Delta h}{h_{\text{stage}}}$$

This becomes, for the compressor:

$$N_c = \frac{\frac{J C_{ps} T_{o1} (r_c^\lambda - 1)}{W} \frac{U^2}{2g} \eta_{sc}}{\frac{U^2}{2g}}$$

and by using (3) to eliminate circumferential speed:

$$N_c = \frac{3112 C_{ps} T_{o1} (r_c^\lambda - 1) (L/d)_1}{\sum_c \eta_{sc} \frac{W}{U^2/2g}} \quad (24)$$

Correspondingly, the number of stages for the turbine is found to be:

$$N_t = \frac{3112 C_{pt} T_{o3} (1 - r_t^\mu) \eta_{st} (L/d)_2}{\sum_t \frac{W}{U^2/2g}} \quad (24a)$$

General experience has shown that the blade aspect ratio, $\delta \equiv L/b$, is in practice limited at the lower extreme by tip losses and wall friction, and at the upper extreme by stress considerations and mounting secondary flow losses. In view of these limits, the aspect ratio may be taken to vary from one to five, with the optimum at possibly two or three for "good practice".

By inserting (8) and (23) in the definition, aspect ratio may be related to other quantities, however, and the result will serve, in combination with the rule-of-thumb above, to limit the range of variation of other parameters. Thus, for the compressor:

$$\delta_{mc} = .0594 \frac{\sin \beta_s}{(C_x/U)_c} \frac{1}{\eta_c^{1/4}} \left[\frac{\sigma_g R T_{o1} (F_{u1} + 1) F_{bc} (L/d)_1}{P_{o1} \sum_c} \right]^{1/2} \quad (25)$$

$$\frac{\sum_{i=1}^n \frac{U_i}{U_i^2} \left(\frac{U_i}{U_i^2} - 1 \right)}{\sum_{i=1}^n \frac{U_i}{U_i^2}}$$

and by using (2) to eliminate the statistical symbol:

$$\frac{\sum_{i=1}^n \frac{U_i}{U_i^2} \left(\frac{U_i}{U_i^2} - 1 \right) \left(\frac{U_i}{U_i^2} - 1 \right)}{\sum_{i=1}^n \frac{U_i}{U_i^2}} \quad (40)$$

Unsurprisingly, the number of stages for the system

is found to be:

$$\frac{\sum_{i=1}^n \frac{U_i}{U_i^2} \left(\frac{U_i}{U_i^2} - 1 \right) \left(\frac{U_i}{U_i^2} - 1 \right)}{\sum_{i=1}^n \frac{U_i}{U_i^2}} \quad (41)$$

General experience has shown that the above

ratio, $\delta \approx 1/2$, is in practice limited by the lower extreme

of slip losses and wall friction, and at the upper extreme by

stress concentrations and resulting secondary flow losses. In

view of these limits, the above ratio may be taken to vary

from one to five, when the degree of possible loss or stress

is 'good practice'.

By inserting (3) and (42) in the definition, suggest

ratio may be related to other quantities, however, and the

result will serve, in combination with the ratio-of-length above,

to limit the range of variation of other parameters. Thus,

for the component:

$$\delta_{\text{comp}} = \frac{\sum_{i=1}^n \frac{U_i}{U_i^2} \left(\frac{U_i}{U_i^2} - 1 \right) \left(\frac{U_i}{U_i^2} - 1 \right)}{\sum_{i=1}^n \frac{U_i}{U_i^2}} \quad (42)$$

and for the turbine:

$$\delta_{nt} = .0594 \frac{\sin \beta_2}{(C_x/U)_t F_t^{1/4}} \left[\frac{\sigma_{RT_{02}} (F_{nt} + r_t^{-1}) F_{bt} (L/d)_t}{P_{04} \Sigma_t} \right]^{1/2} \quad (25a)$$

and for the machine:

$$\delta = \frac{1}{2} \left[\frac{0.0001 \times 10^6}{10^6} \right] = 0.00005$$

V. ROTOR SIZE AND WEIGHT

Rotor size is controlled by the number of stages, the axial length of each, and the tip diameter of the blading. For the length:

$$L^* = 2Y \cdot b_m \cdot N$$

wherein the clearance ratio between adjacent rows is indicated by Y. From (23) and (24), and by properly arranging terms, it may be shown that for the compressor:

$$L_c^* = 29,600 \frac{C_{pc} T_{01} (C_x/U)_1}{\frac{W}{U^*/2g} \eta_{sc}} \left[\frac{Y(L/d)_1}{\sin \beta_a \sigma_g^{\frac{1}{2}} \sum_c^{\frac{3}{4}} F_{bc}^{\frac{1}{2}}} \right] \left[\left(\frac{W}{P^*} \right)^{\frac{1}{2}} F_c^{\frac{1}{2}} (r_c^\lambda - 1) \right] P^{*\frac{1}{2}} \quad (26)$$

and for the turbine:

$$L_t^* = 29,600 \frac{C_{pt} T_{02} (C_x/U)_2 \eta_{st}}{\frac{W}{U^*/2g}} \left[\frac{Y(L/d)_2}{\sigma_g^{\frac{1}{2}} \sum_t^{\frac{3}{4}} F_{bt}^{\frac{1}{2}} \sin \beta_a} \right] \left[\left(\frac{W}{P^*} \right)^{\frac{1}{2}} F_t^{\frac{1}{2}} (1 - r_t^\lambda) \right] P^{*\frac{1}{2}} \quad (26a)$$

In each of the above, the final bracket represents the explicit influence of pressure ratio while the first incorporates the influence of cascade geometry, broadly speaking. The trend of this pressure ratio influence is shown in Fig. 4 for compressor and turbine separately.

Rotor volume may be stated simply as:

$$V = \frac{1}{4} \pi D^2 L^*$$

Substituting from (9) and (26) and rearranging as for the length yields, for the compressor:

$$V_c = \frac{1850 RC_{pt} T_{o1}^3 P^{*3/2}}{\eta_{sc} \frac{W}{U^2/2g} P_{o1} (C_x/U)_1} \left[\frac{Y(1-(L/d)_1)^2 (L/d)_1^{3/4}}{\sin \beta_2 \sigma_g^{\frac{1}{2}} F_{bc}^{\frac{1}{2}} \Sigma_c^{5/4}} \right] \cdot \left[(F_{m1}+1)(r_c^{\lambda}-1) F_c^{\frac{1}{4}} \left(\frac{W}{P^*} \right)^{\frac{3}{2}} \right] \quad (27)$$

and likewise for the turbine:

$$V_t = \frac{1850 RC_{pt} T_{o2}^3 P^{*3/2}}{\frac{W}{U^2/2g} P_{o2} (C_x/U)_2} \eta_{st} \left[\frac{Y(1+(L/d)_2)^2 (L/d)_2^{3/4}}{\sigma_g^{\frac{1}{2}} F_{bt}^{\frac{1}{2}} \sin \beta_2 \Sigma_t^{5/4}} \right] \cdot \left[(F_{m2}+r_t^{\lambda})(1-r_t^{\lambda}) F_t^{\frac{1}{4}} \left(\frac{W}{P^*} \right)^{\frac{3}{2}} \right] \quad (27a)$$

The two brackets here, as in the length relations, are intended to indicate the influence of stage geometry and pressure ratio respectively. The latter are shown in Fig. 4.

Rotor weight has been investigated for wheels of the DeLaval type by LaValle and Huppert (ref. 5). Their results, with some simplifying assumptions to fix a few minor influence variables which have small ranges, are here applied to both turbine and compressor, since the latter is taken to be of disk type construction and therefore comparable in each stage to a DeLaval wheel. All this is predicated on having a maximum allowable blade stress, as expressed by (3).

$$\left[\frac{\sum_{i=1}^N \frac{1}{\sigma_i^2} \left(\frac{1}{\sigma_i^2} - 1 \right) \right] \frac{\sum_{i=1}^N \frac{1}{\sigma_i^2} \left(\frac{1}{\sigma_i^2} - 1 \right)}{\sum_{i=1}^N \frac{1}{\sigma_i^2} \left(\frac{1}{\sigma_i^2} - 1 \right)} = 1$$

(VII) $\left[\left(\frac{1}{\sigma^2} - 1 \right) \frac{1}{\sigma^2} \right]$

and likewise for the variance:

$$\left[\frac{\sum_{i=1}^N \frac{1}{\sigma_i^2} \left(\frac{1}{\sigma_i^2} - 1 \right) \right] \frac{\sum_{i=1}^N \frac{1}{\sigma_i^2} \left(\frac{1}{\sigma_i^2} - 1 \right)}{\sum_{i=1}^N \frac{1}{\sigma_i^2} \left(\frac{1}{\sigma_i^2} - 1 \right)} = 1$$

(VIII) $\left[\left(\frac{1}{\sigma^2} - 1 \right) \frac{1}{\sigma^2} \right]$

For two degrees of freedom, as in the simple case, the variance is indicated to indicate the influence of stage property and variance ratio respectively. The latter are shown in Fig. 1.

Notes on the data presented for models of the
behavior of the blade and the
 also some simplifying assumptions for the case of
 the case which have been made, are not
 in the case of the case, since the latter is based on
 as of the case of the case and therefore comparison is made
 where to a degree of. All this is presented on page
 a certain allowable blade stress, as expressed by (5).

For one wheel or stage the weight is then:

$$\begin{aligned}
 W_s^* &= (\text{disk} + \text{rim} + \text{blades}) \\
 &= \frac{\pi(L/d)^2 d^2 \rho_b}{\delta} \left[\left(\frac{.55t\delta}{(L/d)^2 \Delta_2} + \frac{\rho_d \Delta_1^2}{\rho_b \Delta_2} \right) \Delta_2 + \frac{\rho_d \Delta_1}{\rho_b \delta} + \frac{.55t\delta}{(L/d)} \right]
 \end{aligned}
 \tag{28}$$

wherein:

$$\Delta_1 \equiv \left[\frac{1}{(L/d)} - 1 - \frac{1}{\delta} \right]$$

$$\Delta_2 \equiv \left[\frac{1}{(L/d)} - 1 - \frac{2}{\delta} \right]$$

$$\Delta_3 \equiv \exp \left[.3 \frac{\rho_b}{\rho_d} \Delta_2^2 (L/d) \right] - 1$$

From Fig. 8 may be obtained Δ_1 and Δ_2 .

The material density ratio, (ρ_b/ρ_d) , will approach unity in the case of a turbine or in the case of a compressor not having light metal blades. With this additional simplification (28) is combined with (8) and (24) to yield:

$$\begin{aligned}
 W_c^* &= \frac{70 C_{pe} T_{01}}{\eta_{so} \frac{W}{U^2/2g} \sum_o^{7/4}} \left[\frac{P^* R T_{01}}{P_{01} (C_x/U)_1} \right]^{3/2} \left[(L/d)_1^{1/4} \frac{W_c^*}{d^2} \right] \\
 &\quad \cdot \left[(r_c^\lambda - 1) \left(\frac{W_c}{P^*} \right)^{3/2} (F_{m1} + 1)^{3/2} \right]
 \end{aligned}
 \tag{28a}$$

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$$\frac{\partial \mathcal{L}}{\partial \mathbf{A}} + \frac{\partial \mathcal{L}}{\partial \mathbf{B}} + \lambda \left(\frac{\partial \mathcal{L}}{\partial \mathbf{A}} + \frac{\partial \mathcal{L}}{\partial \mathbf{B}} \right) = \frac{\partial \mathcal{L}}{\partial \mathbf{A}} + \frac{\partial \mathcal{L}}{\partial \mathbf{B}} + \lambda \left(\frac{\partial \mathcal{L}}{\partial \mathbf{A}} + \frac{\partial \mathcal{L}}{\partial \mathbf{B}} \right)$$

1110

$$\frac{1}{2} = \frac{1}{2}$$

$$\left[\frac{1}{2} - \frac{1}{2} \right] = \frac{1}{2} \quad \text{and} \quad \frac{1}{2} = \frac{1}{2}$$

$$I = \left[\frac{(2\pi)^2}{9} \frac{d^2}{d\theta^2} \right]_{\theta=0} \quad \text{or} \quad \frac{2\pi^2}{9} \frac{d^2}{d\theta^2} \quad \text{at } \theta = 0$$

From 1916 to 1917, I was in charge of the

The material is very hard, (4 1/2) will scratch with

Having light cast upon this additional simplification in the case of a particle or in the case of a composite not

(S2) is obtained from (S1) and (R) after substituting (R2).

$$\left[\begin{array}{c} \frac{1}{2} \\ \frac{1}{2} \\ \frac{1}{2} \end{array} \right] \cdot \left[\begin{array}{c} 1 \\ 1 \\ 1 \end{array} \right] = \left[\begin{array}{c} 1 \\ 1 \\ 1 \end{array} \right]$$

(105) $\left[\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \right]$

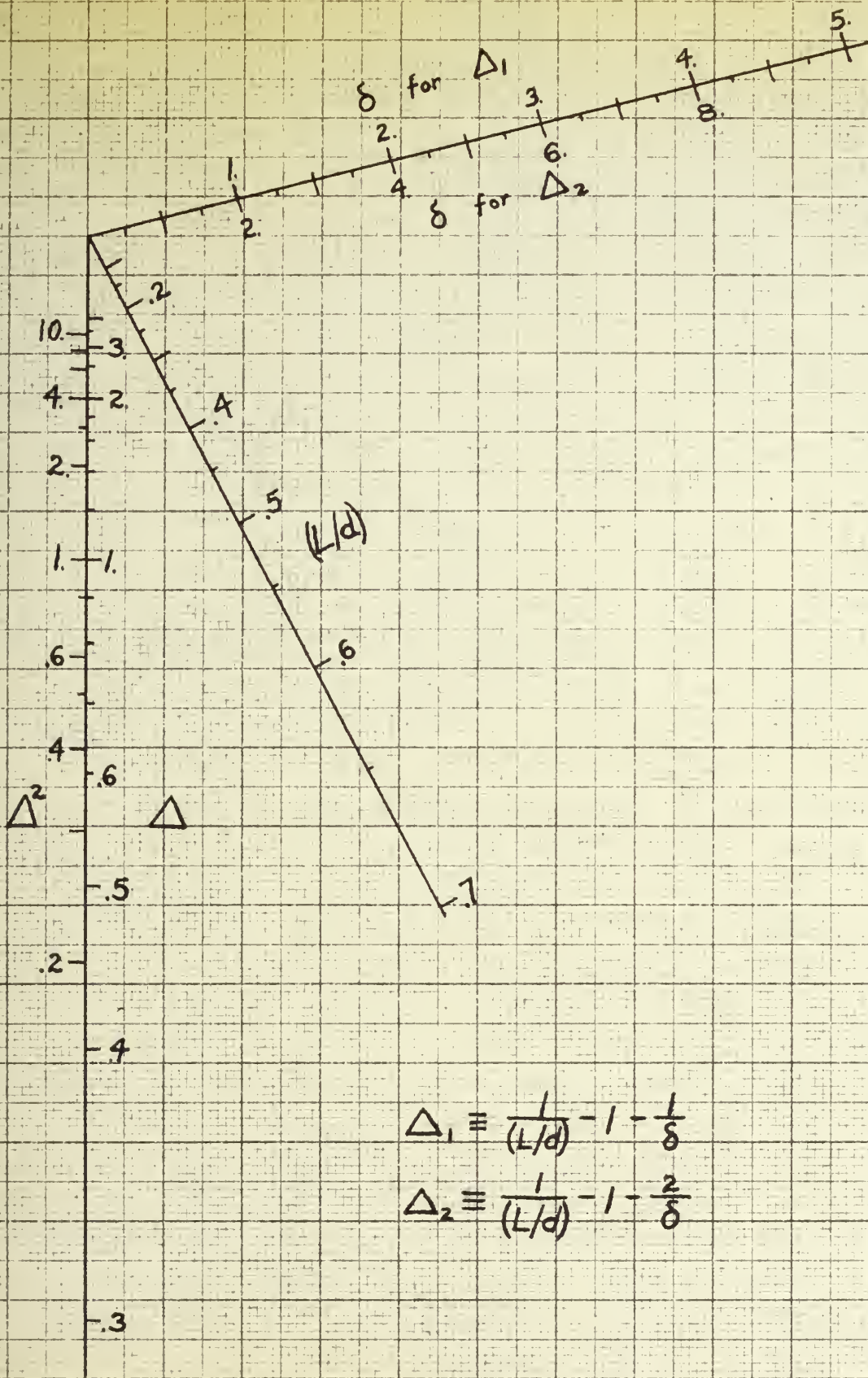


Fig. 8

and:

$$W_t^* = \frac{70 C_{pt} T_{cs} \eta_{st}}{\frac{W}{U^2/2g} \sum_t^{7/4}} \left[\frac{P^* R T_{cs}}{P_{04} (C_x/U)_4} \right]^{3/2} \left[(L/d)_4 \frac{W_s^*}{d^2} \right] \cdot \left[(1-r_t^{-1/2}) \left(\frac{W}{P^*} \right)^{3/2} (F_{m3+r_t}^{-1/2}) \right] \quad (28b)$$

Again the parts have been arranged so that the final bracket in each equation represents the explicit pressure ratio influence and the bracket immediately preceding represents the stage geometry. The pressure ratio factor is shown in Fig. 4 and the geometry factor, calculated from (28) above, appears in Fig. 9.

Due to its effect on rotor thickness, increasing aspect ratio causes a decrease in rotor weight, but a simultaneous rise in rotor volume. This last effect will appear later (see p. 43, eq. (33) et seq.). Hence the mean density of the rotor bulk drops doubly fast with rising aspect ratio. Since a high machine density tends toward compactness of plant for a fixed power output, this gives the first indication that low blade aspect ratios are to be preferred.

$$\frac{V_1}{V_2} = \frac{\sum_{i=1}^N \frac{V_i}{V_1} \cdot \frac{V_i}{V_2}}{\sum_{i=1}^N \frac{V_i}{V_1} \cdot \frac{V_i}{V_2}}$$

$$\left[\frac{V_1}{V_2} \cdot \frac{V_1}{V_2} \cdot \frac{V_1}{V_2} \right]$$

(100)

Again the results have been arranged so that the final pressure in each equation represents the explicit pressure ratio in-
fluence and the bracketed immediately preceding represents the
effect geometry. The pressure ratio factor is shown in Fig. 4
and the geometry factor, calculated from (100) above, appears
in Fig. 5.

Due to the effect on rotor thickness, increasing
slip ratio causes a decrease in rotor weight, but a
simultaneous rise in rotor volume. This last effect will
appear later (see p. 43, eq. (32) at end.). Hence the mean
density of the rotor will drop doubly fast with rising slip
ratio. Since a high machine density tends toward compactness
of plant for a fixed power output, this gives the direct
indication that low blade aspect ratios are to be preferred.

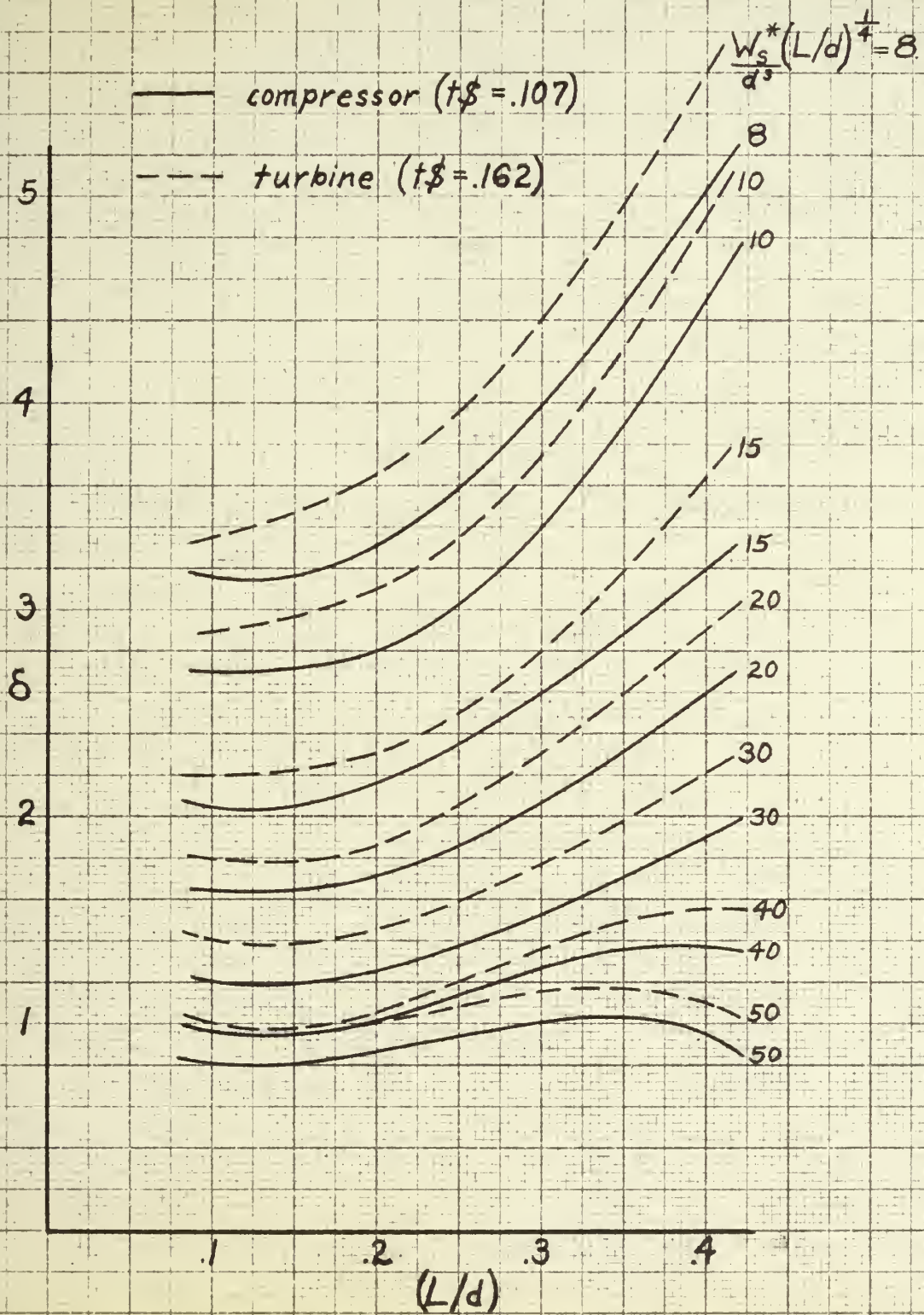


Fig. 9

VI. COMBUSTION COMPONENTS AND CASINGS

Over the variety of combustion turbine plant types, from heavy stationary generating equipment to light weight air-craft propulsion sets, combustion equipment varies in weight and size fully as much as does any other component, and with considerably greater empiricism. Watson and Clarke (ref. 6) have summarized current practice. The first parameter to be considered is heat release rate per unit volume:

$$\frac{h_f w_f}{V_b P_{02}} = \frac{w C_{pb} (T_{02} - T_{01})}{V_b r_c P_{01}} = .65 \frac{\text{BTU}}{\text{sec. lb. ft.}}$$

The constant employed is based on the fact that good combustor performance regularly can be attained for heat release rates up to but not much exceeding 5.7×10^6 BTU/hr. ft.³ atm. The actual limit depends on a balance between heat conduction, diffusion, wall cooling and metallurgical properties. A slightly more conservative figure, 4.95, is used above.

Another limitation is flame stability in the moving gas flow. Based only on experience and good practice again, the permissible maximum bulk velocity entering the flame zone is found to be about 500 ft./sec. Hence, leaving again a margin of safety by using 400, there is:

$$\frac{w}{\rho_2 A_b} = 400 = \frac{w R T_{01} (F_{m1} + r_c \lambda)}{A_b r_c P_{01}} \frac{\text{ft.}}{\text{sec.}}$$

Combustor length is then, assuming a shape approximately prismatic:

Over the range of combustion turbine plant types, from heavy stationary gas-turbine engines to light weight aircraft propulsion units, combustion weight varies in weight and also with as much as 10% of other components, and with considerably greater variation. Nelson and Gilling (ref. 6) have summarized engine trends. The first parameter to be considered is heat release rate per unit

volume:

$$\frac{W}{V} = \frac{W_{D(100-100)} \cdot 100}{V_{D(100-100)} \cdot 100} = 0.84 \text{ sec. lb. ft.}^3$$

The constant supplied is based on the fact that good compressor performance regularly can be obtained for heat release rates of 10 and not much exceeding 2.0 x 10⁶ Btu/hr. ft.³ etc. The actual limit depends on a balance between heat conduction, radiation, wall cooling and metallurgical properties. A slightly more conservative figure, 1.85, is used above.

Another limitation is flame stability in the moving gas flow. Based only on experience and good practice again, the permissible maximum wall velocity entering the flame zone is found to be about 500 ft./sec. Hence, leaving again a margin of safety of being 400, there is:

$$\frac{W}{V} = 400 = \frac{W_{D(100-100)} \cdot 100}{V_{D(100-100)} \cdot 100} = 0.84$$

Compressor length is then, assuming a shape approximately planar

$$L_b^* = \frac{V_b}{A_b} = 616 \frac{C_{pb} [(T_{02}/T_{01}) - r_c^\lambda]}{R(F_{m1} + r_c^\lambda)} \text{ ft.} \quad (29)$$

In case, as is true for aircraft and certain other plants, combustion is to be equally shared among several chambers, a third limiting factor to be considered is minimum flame tube diameter. This may be met by: (a) limiting the number of separate chambers; or (b) increasing the total cross-sectional area beyond that required for permissible maximum gas velocity.

The weight of a combustor is a function of wall construction and surface area. Assuming that if a number of chambers are operated in parallel they are all exactly alike, then the total surface area is:

$$\begin{aligned} A_s &= \pi d_b L^* N = 2 V_b (\pi N / A_b)^{1/2} \\ &= 109 C_{pb} \left[\frac{T_{02}}{T_{01}} - r_c^\lambda \right] \left[\frac{N W T_{01}}{R r_c P_{01} (F_{m1} + r_c^\lambda)} \right]^{1/2} \text{ ft.}^2 \end{aligned}$$

Wall construction is considered to be based on resistance to sagging and buckling rather than to rupture in tension. Thus for a cylinder in transverse loading, as a first approximation,

$$\begin{aligned} \sigma &= \frac{my}{I} \sim \frac{(t.d)d}{(t.d)^3} \sim \frac{d}{(t.d)^2} \\ W_b^* &\sim A_s (t.d) \sim A_s (d/\sigma)^{1/2} = A_s d / (\sigma d)^{1/2} \end{aligned}$$

But since

$$A_b \sim d^3 N \quad \text{and} \quad V_b \sim d^2 N L^* \quad \text{there follows:}$$

$$C_p = \frac{1}{2} \rho V^2 \frac{C_{Dp}}{C_{D0}} \quad (22)$$

In case, as is true for aircraft and certain other planar, operation is to be equally varied among various channels, a fixed limiting factor to be maintained in relation to the total weight. This can be set by (a) limiting the number of separate channels or (b) increasing the total cross-sectional area beyond that required for parasitic resistance for velocity.

The weight of a conductor is a function of wall construction and surface area. Assuming that if a number of channels are operated in parallel that are all equally sized, then the total surface area is:

$$A_p = 2\pi r L \sqrt{1 + \left(\frac{dr}{dz} \right)^2} = 2\pi r L \sqrt{1 + \left(\frac{dr}{dz} \right)^2}$$

Wall construction is considered to be based on resistance to weight and similar weight loss in response to tension. Thus for a cylinder in transverse loading, as a first approximation,

$$\sigma = \frac{W}{A} \sim \frac{1}{2} \rho V^2 \frac{C_{Dp}}{C_{D0}} \quad (23)$$

and $\sigma \sim \frac{1}{2} \rho V^2$ and $\sigma \sim \frac{1}{2} \rho V^2$ to be followed:

$$W_b^* \sim \frac{V_b}{(\sigma_d)^{1/2}} \sim V_b \left[\frac{N}{\sigma^2 A_b} \right]^{1/4}$$

In order to eliminate the tensile stress from this expression the proportionality $\sigma \sim T_{03}^{-4}$, obtainable from Fig. 10, to give:

$$W_b^* \sim C_{pb} T_{03}^2 \left[\frac{T_{03}}{T_{01}} - r_c^\lambda \right] \left[\frac{W T_{01}}{r_c P_{01}} \right]^{3/4} \left[\frac{N}{R(F_{m1} + r_c^\lambda)} \right]^{1/4}$$

The foregoing was intended to apply to metallic combustors only. Another type, intended for large marine or stationary plants and lined with refractory, requires separate treatment.

Data on current combustion equipment permit approximate evaluation of the constant of proportionality to give:

$$W_b^* = \frac{C_{pb} T_{03}^2}{2000} \left[\frac{T_{03}}{T_{01}} - r_c^\lambda \right] \left[\frac{W T_{01}}{r_c P_{01}} \right]^{3/4} \left[\frac{N}{R(F_{m1} + r_c^\lambda)} \right]^{1/4} \quad (30)$$

Further, the space required for a combustion system composed of a set of identical can-type units, as opposed to the net internal or gas volume used above, may be approximated as:

$$\begin{aligned} V_b^* &= \pi (1.5 d_b)^2 L_b^* \\ &= \frac{725 N d_b^2 C_{pb}}{R} \cdot \frac{(T_{03}/T_{01}) - r_c^\lambda}{(F_{m1} + r_c^\lambda)} \end{aligned}$$

By making use of the flame cross-sectional area relation for proper gas velocity mentioned at the beginning of this section, this becomes:

1/4

$$\left[\frac{E}{\sigma_{\text{max}}} \right] \sim \frac{1}{\sigma_{\text{max}}} \left[\frac{E}{\sigma_{\text{max}}} \right]$$

In order to eliminate the possible effects from this approximation the proportionality σ_{max}^{-1} is obtainable from

Fig. 10, we have:

$$\frac{E}{\sigma_{\text{max}}} \sim \frac{1}{\sigma_{\text{max}}} \left[\frac{E}{\sigma_{\text{max}}} \right] \left[\frac{1}{\sigma_{\text{max}}} \right] \left[\frac{1}{\sigma_{\text{max}}} \right] \left[\frac{1}{\sigma_{\text{max}}} \right]$$

The foregoing was intended to apply to metallic components only. Another type, intended for large marine or stationary plants and lined with refractory, requires separate treatment.

Due to certain constructional requirements herein specified, the evaluation of the constant of proportionality is given:

$$\frac{E}{\sigma_{\text{max}}} \sim \frac{1}{\sigma_{\text{max}}} \left[\frac{E}{\sigma_{\text{max}}} \right] \left[\frac{1}{\sigma_{\text{max}}} \right] \left[\frac{1}{\sigma_{\text{max}}} \right] \left[\frac{1}{\sigma_{\text{max}}} \right] \quad (20)$$

Further, the stress reduction for a combustion system composed of a set of identical con-type units, as opposed to the use of a single unit, may be approximated by the use of the following:

$$\frac{E}{\sigma_{\text{max}}} \sim \frac{1}{\sigma_{\text{max}}} \left[\frac{E}{\sigma_{\text{max}}} \right] \left[\frac{1}{\sigma_{\text{max}}} \right] \left[\frac{1}{\sigma_{\text{max}}} \right] \left[\frac{1}{\sigma_{\text{max}}} \right]$$

By using one of the stress cross-sectional area reduction for proper gas velocity reduction at the beginning of this section, this becomes:

Stress to Cause Creep Rate of
1. % per 10,000 hrs.

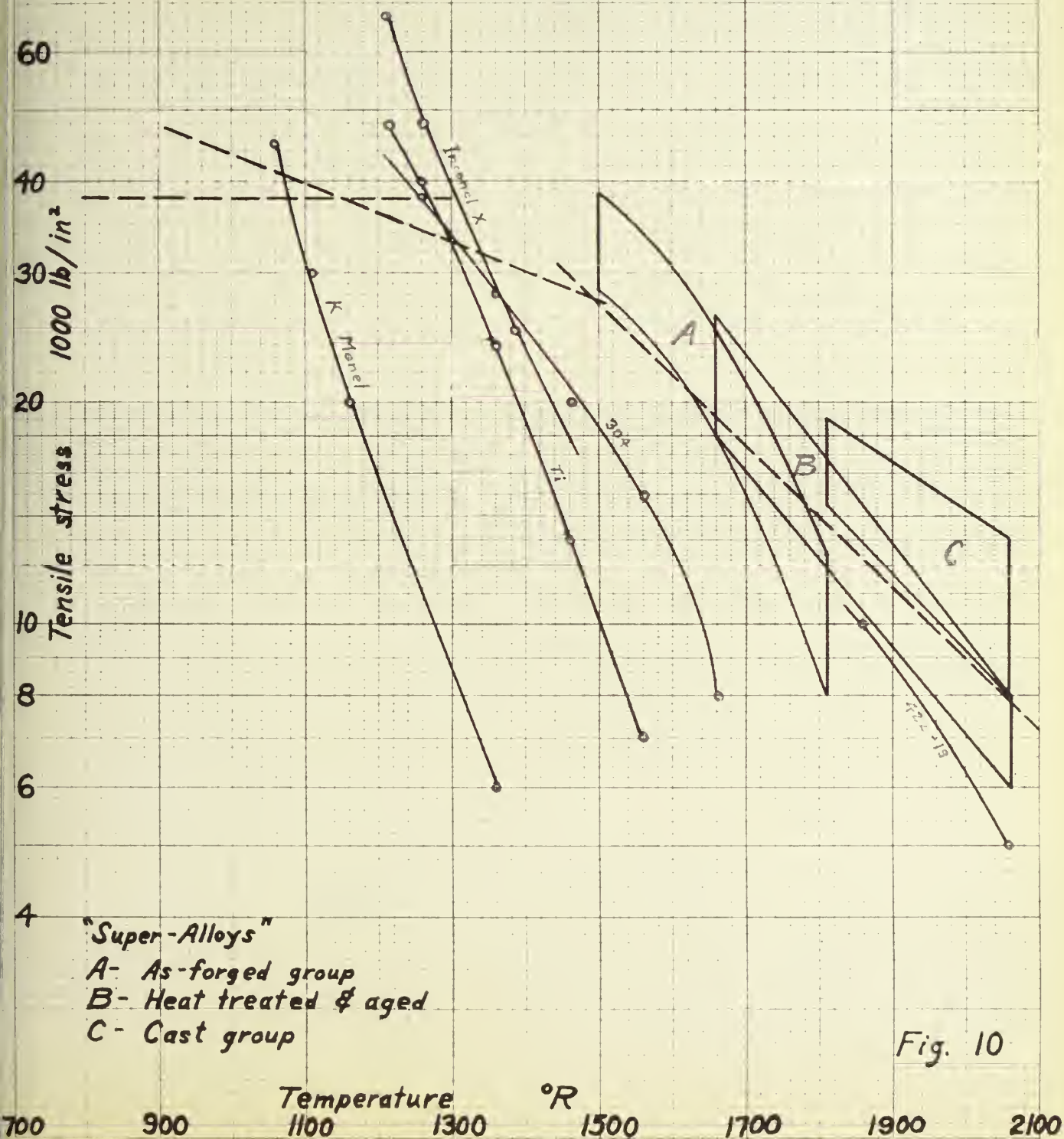


Fig. 10

$$V_b^* = 2.31 \frac{C_{pb} T_{01} P^*}{P_{01}} \left[\frac{(T_{03}/T_{01}) - r_c^\lambda}{r_c} \left(\frac{w}{P^*} \right) \right] \quad (31)$$

The trend of combustor volume with pressure ratio as given by this is shown for the single temperature ratio $(T_{03}/T_{01}) = 3.7$ in Fig. 4.

The construction of stators and casings is considered to be governed by regard for stiffness rather more than for rupture strength. Stator blading weight is assumed to average the same as that of rotor blading operating within it. Following ref. 5 and the foregoing treatment of combustor shell weight, the stator blading and casing weights become:

$$W^* = \frac{\pi(L/d)^2 d^3 \rho_b}{\delta} \left[\frac{t\beta}{2(L/d)} \right] \quad \text{for the blades}$$

$$W^* \sim V_r^* \left[\frac{1}{\sigma^* D_r^2} \right]^{1/4} \sim d_r^{3/2} [1 - (L/d)]^{3/2} L_r^* \quad \text{for the casing}$$

Again the constant is approximated on the basis of current plants to give, for the compressor:

$$W_{cc}^* = \pi d_c^{3/2} \rho_b \left[\frac{t\beta(L/d)_c d_c^{3/2}}{2\delta} + [0.06 \, 1 + (L/d)_c]^{3/2} L_c^* \right] \quad (32)$$

and for the turbine:

$$W_{tc}^* = \pi d_t^{3/2} \rho_b \left[\frac{t\beta(L/d)_t d_t^{3/2}}{2\delta} + .01 [1 + (L/d)_t]^{3/2} L_t^* \right] \quad (32a)$$

both of which increase monotonically with (L/d) .

Equally defensible is the approximation to casing volume as, in the case of the compressor:

$$V_{cc}^* = V_c \left[\frac{D+2b}{D} \right]^3 = V_c (1+2b/D)^3$$

$$(22) \quad \left[\left(\frac{2}{5} \right) - \frac{(1.5 - 1.25)}{1.5} \right] \frac{1.5 - 1.25}{1.5} = 0.05$$

The mean of estimator values with respect to θ .

...and the time is spent in the most efficient manner.

... 327 ... (10/10)

beating up at anyone has made to contribute all

to be covered by means of different rubber boots for

Report of Bureau of Prisons, Washington, D.C.

the use as that of a normal bloodless operation with it.

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about the state's fishing and boating season;

$$\frac{d}{dt} \left(\frac{1}{2} m v^2 \right) = \frac{1}{2} m \frac{d}{dt} (v^2) = \frac{1}{2} m \frac{d}{dt} (v_x^2 + v_y^2 + v_z^2)$$

$$V_{\text{eff}} = \frac{1}{2} \left[\frac{1}{2} \left(\frac{1}{2} \right) \right]$$

It is noted that the following information is contained in the file:

...and please to give, for the Government:

$$\left[\frac{1}{2} \left(\frac{1}{2} \right) + \frac{1}{2} \right] + \frac{1}{2} \left(\frac{1}{2} \right) = \frac{1}{2}$$

4 80:10-11:00 and 12:00 10 3

$$\left[\frac{1}{2} \left(\frac{1}{\omega} + 1 \right) \right] \frac{1}{2} + \frac{1}{2} \left(\frac{1}{\omega} + 1 \right) \frac{1}{2}$$

body of solid increase exponentially with (λ/λ_0) .

There is an approximation to the

Volume 23, in the case of the defendant:

10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80	81	82	83	84	85	86	87	88	89	90	91	92	93	94	95	96	97	98	99	100
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but since

$$\delta = (L/d) (d/b) \quad \text{and} \quad b/D = (b/d)(d/D)$$

by (9) there appears that:

$$v_{cc}^* \approx v_c \left[1 + 6b/D \right] = v_c \left[1 + \frac{(6/\delta_{mc})(L/d)_1}{1 + (L/d)_1} \right] \quad (33)$$

and similarly for the turbine casing volume.

$$v_{tc}^* = v_t \left[1 + \frac{(6/\delta_{mt})(L/d)_4}{1 + (L/d)_4} \right] \quad (33a)$$

$$(c\backslash d)(d\backslash c) = a\backslash b$$

$$(c\backslash d)(d\backslash c) = \delta$$

by (2) these operators are:

$$\left[\frac{(c\backslash d)(d\backslash c)}{(c\backslash d)+1} \right]_0^{\infty} = \left[\frac{(c\backslash d)(d\backslash c)}{(c\backslash d)+1} \right]_0^{\infty} \approx \frac{1}{2}$$

and similarly for the other two cases.

$$\left[\frac{(c\backslash d)(d\backslash c)}{(c\backslash d)+1} \right]_0^{\infty} = \frac{1}{2}$$

(56)

(57)

SCROLLS, VOLUTES and DUCTS

Scrolls, volutes, ducts, piping, stacks and so on vary greatly from plant to plant for the good reason that plant location and job assignment dictate component layout to a large extent. The straight-through aircraft jet engine with axial compressor certainly has the minimum of such parts, whereas a marine propulsion or shore power generation plant has a good deal of its total weight and space so constituted. Hence no definitive mathematical treatment can be shown, but a few general rules can be given.

For the simple cycle, gas pressures up to the compressor inlet and beyond the turbine exit are of the order of one atmosphere. Within these sections of the flow path design is considered to be based on rigidity rather than bursting pressure. Hence, following the reasoning of ref 9:

$$V^* \sim w$$

$$W^* \sim w$$

From the compressor discharge to the turbine nozzle the flow path carries the same weight flow at a pressure r_0 times as great and a specific volume correspondingly less. In this section design is considered to be based on bursting strength, but in the first approximation it turns out again that:

$$W^* \sim w$$

while

$$V^* \sim w/r_0$$

ROCKETS, VOLUTES AND DUCTS

Volutes, volutes, ducts, pipes, tanks and so on very greatly from plant to plant for the good reason that plant location and job arrangement dictate component layout to a large extent. The through-through aircraft jet engine with axial compressor certainly has the minimum of such parts, whereas a turbine propulsion or short power generation plant has a good deal of its total weight and space so constituted. Hence no definitive mathematical treatment can be shown, but a few general rules can be given. For the single cycle, the compressor up to the compressor inlet and beyond the turbine exit are at the center of the atmosphere. Within these sections of the flow path design is considered to be based on rigidity rather than existing materials. Hence, following the reasoning of ref 9:

$$V \sim W$$

$$V \sim W$$

From the compressor inlet to the turbine inlet the flow path carries the same weight flow at a pressure 1.0 times as great and a specific volume correspondingly less. In this section design is considered to be based on weight strength, but in the first approximation it turns out again that:

$$V \sim W$$

$$V \sim W^{1/2}$$

while

Since it has been shown that the net power output for given cycle conditions is:

$$P^* \sim w$$

then it may be concluded that for a given type of plant -- aircraft, mobile or stationary -- the size and volume of ducting will vary nearly linearly with the mass flow, once the cycle thermodynamic conditions are fixed. Taking account of the relation between air rate and pressure ratio, the weight and size of ducting and connections may be expressed as a function of the variable (w/P^*) to fit any plant capacity.

Since it has been shown that the net power output

for given cycle conditions is:

$$P \sim W$$

then it may be concluded that for a given type of plant --

aircraft, mobile or stationary -- the size and volume of

ducting will vary nearly linearly with the mass flow, once

the cycle thermodynamic conditions are fixed. Taking ac-

count of the relation between air rate and pressure ratio,

the weight and size of ducting and connections may be ex-

pressed as a function of the variable (W/P^*) to fit any

plant capacity.

VII. APPLICATION

In order to clarify the effect of certain variables as they individually influence the design, it is advisable to fix as many of the other variables as possible, or in other words to take a particular plant and inquire as to how the internal geometry of the components may be optimized to produce the most power for the least investment.

Choosing a simple CBTG plant (Fig. 1) for this purpose, the following points are fixed:

<u>Compressor</u>	<u>Turbine</u>
$T_{o1} = 530^{\circ}\text{R}$	$T_{o3} = 1960^{\circ}\text{R}$
$C_p = .24$	$C_p = .27$
$\eta_{so} = .90$	$\eta_{sc} = .85$
$\beta_s = 60^{\circ}$	$\beta_s = 30^{\circ}$
$\theta = 40^{\circ}$	$\theta = 60^{\circ}$
$t = .10$	$t = .15$
$\sigma_b = 22,000 \text{ psi}$	$\sigma_b = 24,000 \text{ psi}$
$\sigma_g = 4000 \text{ psi}$	$\sigma_g = 5000 \text{ psi}$
Both machines:	
$r = 5$	$\gamma = .7$
$R = 53.3 \text{ ft./}^{\circ}\text{R}$	$\rho_b = \rho_d = 500 \text{ lb./ft.}^3$
$Y = 1.3$	$P_{o1} = P_{o4} = 2116 \text{ lb./ft.}^2$

VII. APPLICATION

In order to clarify the effect of certain variables on the radial inflow turbine, it is advisable to fix as many of the other variables as possible, or in other words to take a particular plant and inquire as to how the internal geometry of the components may be optimized to produce the most power for the least investment. Choosing a simple OGTG plant (Fig. 1) for this purpose, the following points are fixed:

<u>Compressor</u>		<u>Turbine</u>	
$T_{01} = 520^{\circ}R$		$T_{03} = 1800^{\circ}R$	
$\eta_c = .84$		$\eta_t = .87$	
$\gamma_{ec} = .30$		$\gamma_{et} = .33$	
$\beta_c = 60^{\circ}$		$\beta_t = 30^{\circ}$	
$\theta = 40^{\circ}$		$\phi = 60^{\circ}$	
$t = .10$		$\tau = .15$	
$\dot{V}_c = 32,000 \text{ cfm}$		$\dot{V}_t = 24,000 \text{ cfm}$	
$\dot{V}_g = 4000 \text{ cfm}$		$\dot{V}_s = 3000 \text{ cfm}$	
Both machines:			
$\gamma = .3$		$\gamma = .3$	
$\beta = 52.5^{\circ}$		$\beta = 50.15^{\circ}$	
$\gamma = 1.3$		$\gamma = 1.3$	

From these it follows that (Fig. 11 et seq):

$$\begin{array}{ll}
 F_{m_1} = .1 & F_{m_2} = .03 \\
 F_{bo} = 2.85 & F_{bt} = 6.58 \\
 \lambda = .318 & \mu = .211 \\
 r_o^\lambda = 1.67 & r_t^\mu = .711 \\
 r_o^{1-\lambda} = 3.0 & r_t^{1-\mu} = 3.56 \\
 F_o = .5 & F_t = .439 \\
 \Sigma_o = 9050. & \Sigma_t = 9870. \\
 w/P^* = .0105 \text{ lb/HP sec} & \\
 \beta_o = .96 & \beta_r = 1.08
 \end{array}$$

Having decided to investigate the effect of the flow coefficient (C_x/U), blade length ratio (L/d) and mass flow (w), all other parameters are expressed in terms of these three by using the information above and the equations set forth previously. That these three variables are independent may be seen from the continuity relation and (3). Under the original simplifying assumptions and the above set conditions none of these three variables affect the thermodynamics of the cycle, however, and so the air rate (w/P^*) remains unchanged. The effect of scaling up the power rating of the plant, then, for any fixed set of values such as the above may be taken directly as the effect of increasing the mass flow rate.

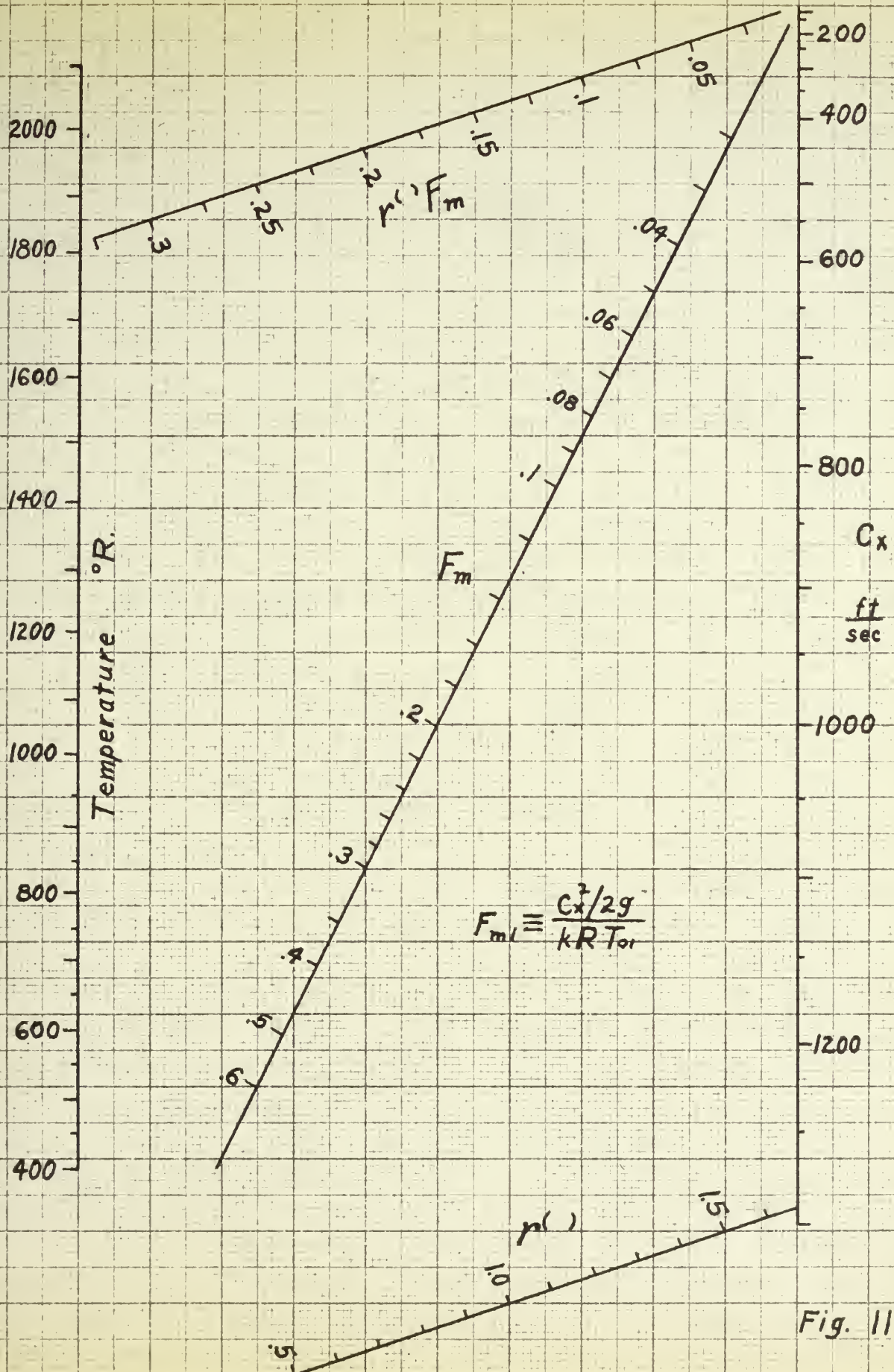
Equations here are numbered to correspond to their counterparts in the preceding general development. They are:

from these it follows that (Fig. 11 at top):

$$\begin{aligned} T_{01} &= 1.1 & T_{02} &= 0.9 \\ T_{03} &= 2.28 & T_{04} &= 0.66 \\ \lambda &= 0.218 & \mu &= 0.211 \\ \gamma &= 1.27 & \gamma^* &= 0.711 \\ \gamma_0 &= 3.0 & \gamma_0^* &= 0.33 \\ \gamma_0 &= 0 & \gamma_0^* &= 0.439 \\ \gamma_0 &= 0.000 & \gamma_0^* &= 0.3370 \\ w/\gamma &= 0.0102 \text{ lb/hr sec} & & \\ \gamma &= 0.99 & \gamma^* &= 1.06 \end{aligned}$$

Having decided to investigate the effect of the flow coefficient ($C_x \sqrt{U}$), blade length ratio (l/b) and mass flow (w), all other parameters are expressed in terms of these three by using the information above and the equations set forth previously. That these three variables are independent may be seen from the continuity relation and (5). Under the original simplifying assumptions and the above set conditions none of these three variables affect the thermodynamic of the cycle, however, and so the air rate (w/γ^*) remains unchanged. The effect of scaling up the power rating of the plant, then, for any fixed set of values such as the above may be taken directly as the effect of increasing the mass flow rate.

Equations here are numbered to correspond to their counterparts in the preceding general development. They are:



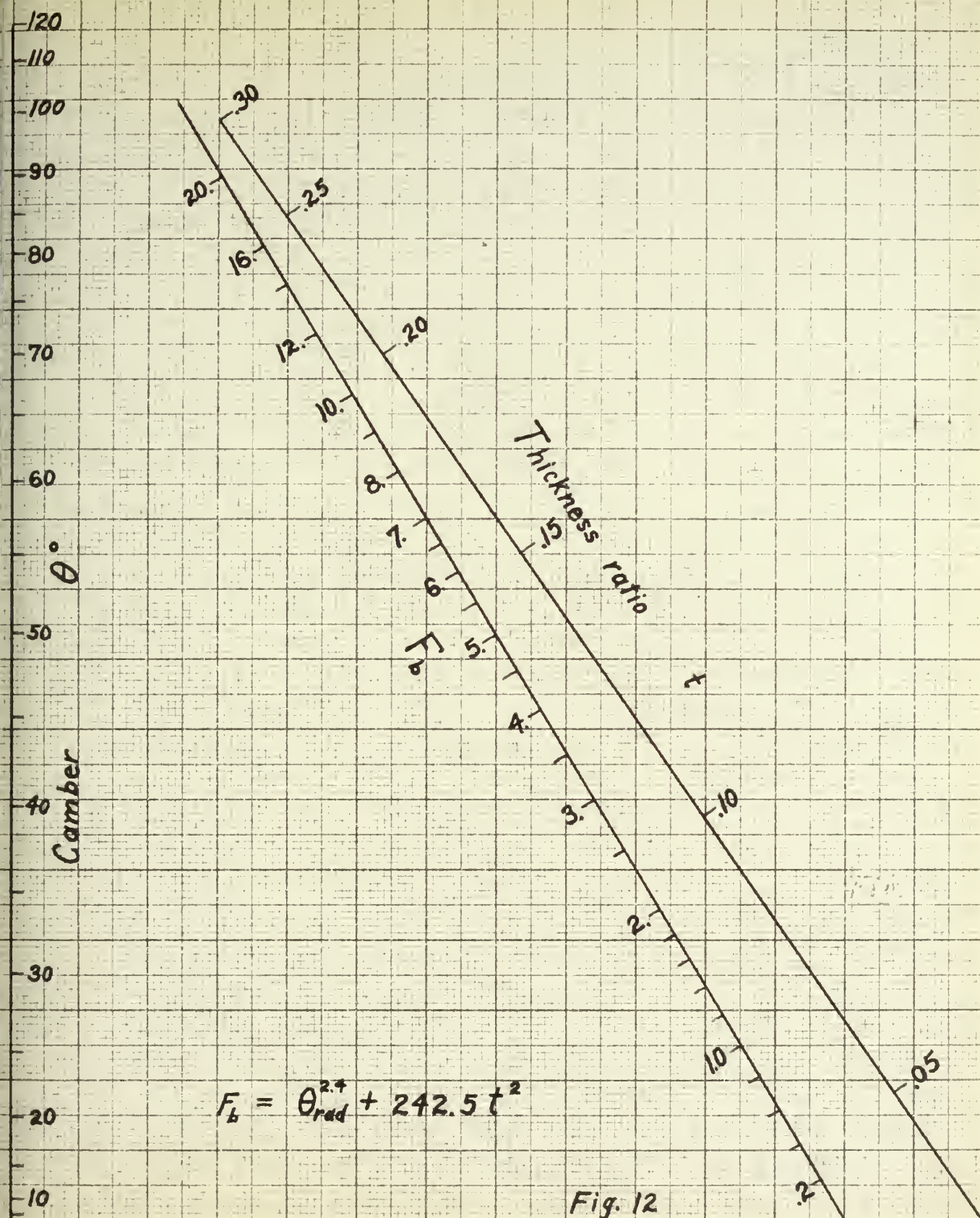


Fig. 12

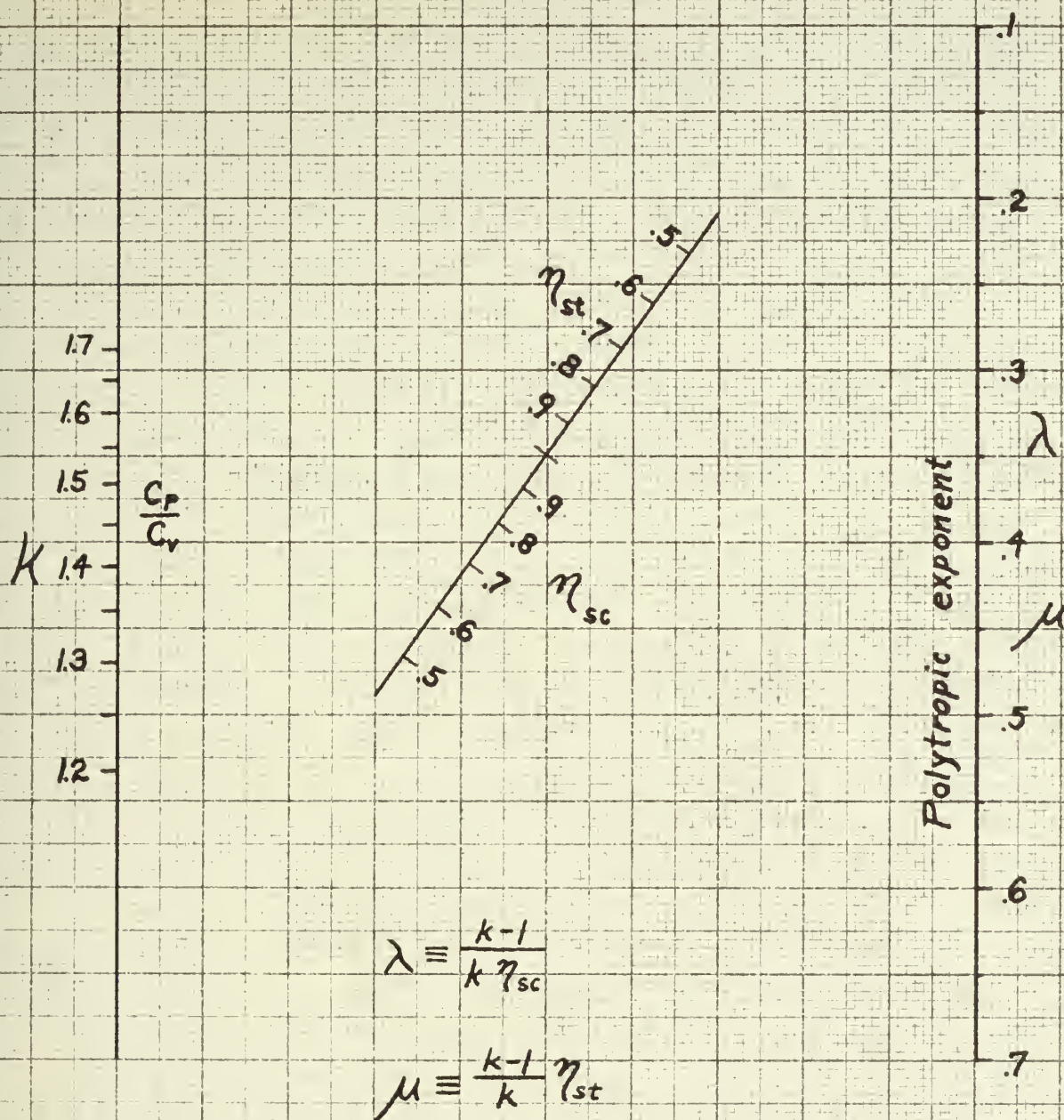


Fig. 13

$$d_o = .1106 \frac{w^{1/2}}{(C_x/U)_1^{1/2} (L/d)_1^{1/4}} \quad (8)$$

$$d_t = .171 \frac{w^{1/2}}{(C_x/U)_4^{1/2} (L/d)_4^{1/4}} \quad (8a)$$

$$G = .674 \left[\frac{(C_x/U)_1}{(C_x/U)_4} \right]^{1/2} \left[\frac{(L/d)_1}{(L/d)_4} \right]^{1/4} \quad (15)$$

$$\delta_{mc} = 3.15 \frac{(L/d)_1^{1/2}}{(C_x/U)_1} \quad (25)$$

$$\delta_{mt} = 4.84 \frac{(L/d)_4^{1/2}}{(C_x/U)_4} \quad (25a)$$

$$\delta_{mc}/\delta_{mt} = 1.43 G^2$$

$$\left[\frac{W}{U^2/2g} \right]_o = 1.024 (C_x/U)_1 \quad (18)$$

$$\left[\frac{W}{U^2/2g} \right]_t = 3.46 (C_x/U)_4 \quad (18)$$

$$v_{oc}^* = .0282 \frac{w^{3/2} (L/d)_1^{3/4}}{(C_x/U)_1^{3/2}} \left[\frac{1.9(C_x/U)_1 (L/d)_1^{1/2}}{1 + (L/d)_1} \right] \left[1 + (L/d)_1 \right]^2 \quad (33)$$

$$v_{tc}^* = .0252 \frac{w^{3/2} (L/d)_4^{3/4}}{(C_x/U)_4^{3/2}} \left[\frac{1.24(C_x/U)_4 (L/d)_4^{1/2}}{1 + (L/d)_4} \right] \left[1 + (L/d)_1 \right]^2 \quad (33a)$$

These last two may also be written:

$$v_{oc}^* = .00504 (w \delta_{mc})^{3/2} \left[\frac{(6/\delta_{mc})(L/d)_1}{1 + (L/d)_1} \right] \left[1 + (L/d)_1 \right]^2$$

$$(2) \quad \frac{W}{r(b \setminus I) \cdot \frac{r(b \setminus I)}{r(b \setminus I)}} = 1.1111 = 1.1111$$

$$(2B) \quad \frac{S \setminus I}{r(b \setminus I) \cdot \frac{r(b \setminus I)}{r(b \setminus I)}} = 1.1111 = 1.1111$$

$$(2C) \quad \frac{r(b \setminus I)}{r(b \setminus I) \cdot \frac{r(b \setminus I)}{r(b \setminus I)}} = 1.1111 = 1.1111$$

$$(2D) \quad \frac{r(b \setminus I)}{r(b \setminus I) \cdot \frac{r(b \setminus I)}{r(b \setminus I)}} = 1.1111 = 1.1111$$

$$(2E) \quad \frac{r(b \setminus I)}{r(b \setminus I) \cdot \frac{r(b \setminus I)}{r(b \setminus I)}} = 1.1111 = 1.1111$$

$$r(b \setminus I) = 1.1111$$

$$(2F) \quad r(b \setminus I) = 1.1111 = \left[\frac{W}{r(b \setminus I)} \right]$$

$$(2G) \quad r(b \setminus I) = 1.1111 = \left[\frac{W}{r(b \setminus I)} \right]$$

$$(2H) \quad \left[\frac{r(b \setminus I)}{r(b \setminus I) + 1} \right] \cdot \frac{r(b \setminus I)}{r(b \setminus I) \cdot \frac{r(b \setminus I)}{r(b \setminus I)}} = 1.1111 = 1.1111$$

$$(2I) \quad \left[\frac{r(b \setminus I)}{r(b \setminus I) + 1} \right] \cdot \frac{r(b \setminus I)}{r(b \setminus I) \cdot \frac{r(b \setminus I)}{r(b \setminus I)}} = 1.1111 = 1.1111$$

These last two may also be written:

$$\left[\frac{r(b \setminus I)}{r(b \setminus I) + 1} \right] \cdot \frac{r(b \setminus I)}{r(b \setminus I) \cdot \frac{r(b \setminus I)}{r(b \setminus I)}} = 1.1111 = 1.1111$$

$$v_{tc}^* = .00237 (w \delta_{mt})^{3/2} \left[1 + \frac{(6/\delta_{mt})(L/d)_4}{1 + (L/d)_4} \right] \left[1 + (L/d)_4 \right]^2$$

$$w_o^* = .0433 \frac{w^{3/2}}{(C_x/U)_1^{5/2}} \left[\frac{1/4}{(L/d)_1} (w_s^*/d_c^3) \right] \quad (28a)$$

$$w_t^* = .057 \frac{w^{3/2}}{(C_x/U)_4^{5/2}} \left[\frac{1/4}{(L/d)_1} (w_s^*/d_c^3) \right] \quad (28b)$$

$$w_{co}^* = \frac{1.02 w^{5/4} (L/d)_1^{7/8}}{(C_x/U)_1^{5/4}} \left[.0318 \frac{(C_x/U)_1^{3/4} w^{1/4}}{(L/d)_1^{1/8}} + \left[1 + (L/d)_1 \right]^{3/2} \right] \quad (32)$$

$$w_{ts}^* = \frac{1.23 w^{5/4} (L/d)_4^{7/8}}{(C_x/U)_4^{5/4}} \left[.108 \frac{(C_x/U)_4^{3/4} w^{1/4}}{(L/d)_1^{1/8}} + \left[1 + (L/d)_4 \right]^{3/2} \right] \quad (32a)$$

$$v_b^* = .0611 w$$

$$w_b^* = 53.8 w^{3/4}$$

From (15), (25) and (25a) above the curves of Fig. 14 have been drawn. As brought out earlier the aspect ratio has been found in practice to be restricted, by loss considerations, to upper and lower limits of about five and one, respectively. Likewise, the blade length ratio (L/d) is restricted at the upper limit by blade stresses and the mechanics of construction, while restricted at the lower limit by

changes of concentration, while restricted at the lower limit by
 situated at the upper limit by plate stresses and the re-
 spective. Likewise, the plate length ratio (l/a) is re-
 stricted, to upper and lower limits of about five and one,
 been found in practice to be restricted, by local considera-
 have been drawn. We brought out earlier the aspect ratio has
 from (12), (13), and (14) above the curves of Fig. 14

$$\frac{w}{b} = 52.5 \times \frac{b}{a}$$

$$\frac{w}{b} = 0.011 w$$

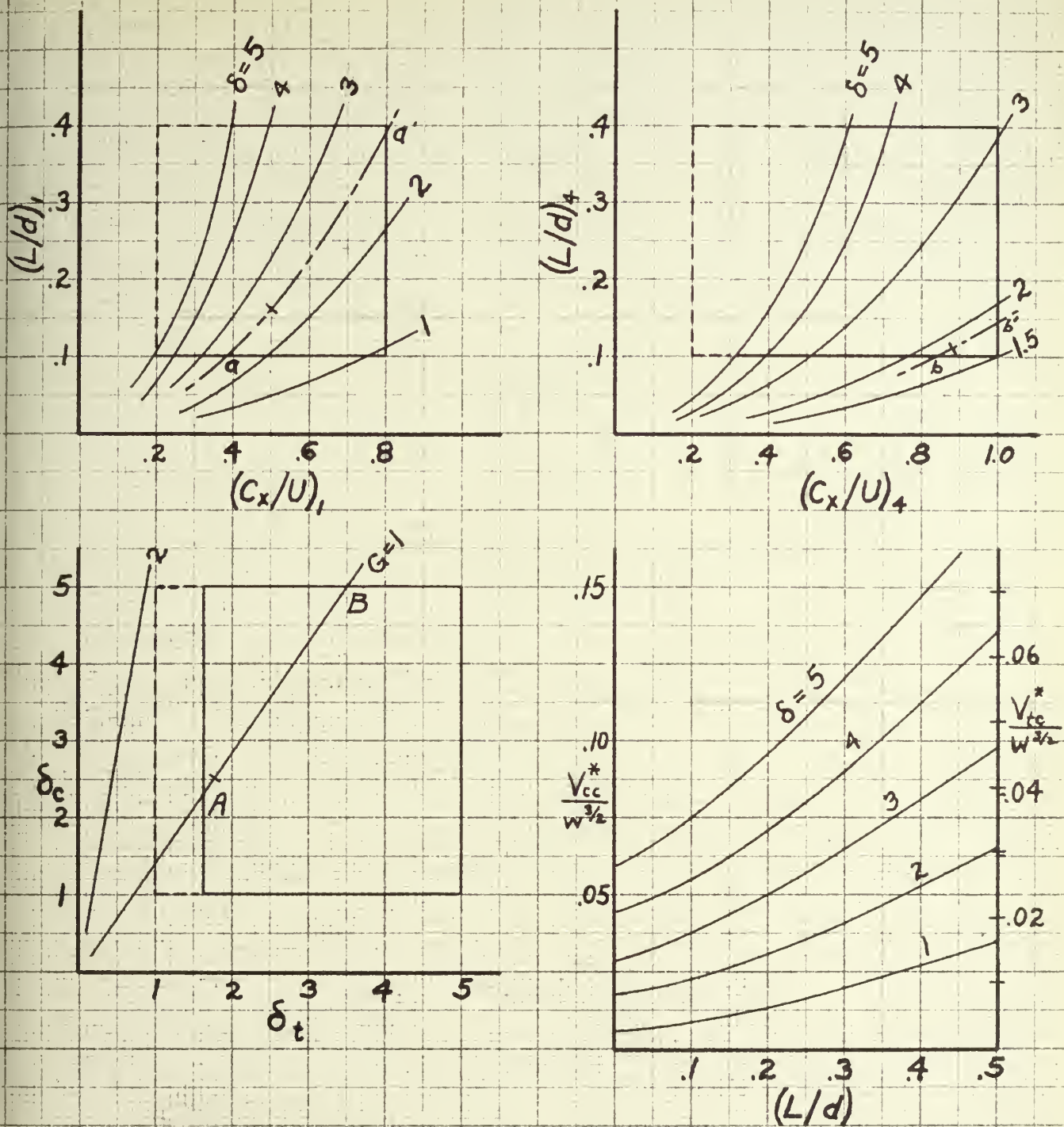
$$\frac{w}{b} = \left[\frac{1.28 w}{(C_X \sqrt{U})^2} \frac{b/a}{(L/a)^2} + \left[\frac{1.08}{(L/a)^2} \frac{(C_X \sqrt{U})^2}{(L/a)^2} + 1 + (L/a)^2 \right] \right] \quad (22a)$$

$$\frac{w}{b} = \left[\frac{1.08 w}{(C_X \sqrt{U})^2} \frac{b/a}{(L/a)^2} + \left[\frac{1.08}{(L/a)^2} \frac{(C_X \sqrt{U})^2}{(L/a)^2} + 1 + (L/a)^2 \right] \right] \quad (22)$$

$$\frac{w}{b} = \left[\frac{1.087}{(C_X \sqrt{U})^2} \frac{b/a}{(L/a)^2} + \left[\frac{1.087}{(L/a)^2} \frac{(C_X \sqrt{U})^2}{(L/a)^2} + 1 + (L/a)^2 \right] \right] \quad (22b)$$

$$\frac{w}{b} = \left[\frac{1.013}{(C_X \sqrt{U})^2} \frac{b/a}{(L/a)^2} + \left[\frac{1.013}{(L/a)^2} \frac{(C_X \sqrt{U})^2}{(L/a)^2} + 1 + (L/a)^2 \right] \right] \quad (22c)$$

$$\frac{w}{b} = \left[\frac{1.00237}{(C_X \sqrt{U})^2} \frac{b/a}{(L/a)^2} + \left[\frac{1.00237}{(L/a)^2} \frac{(C_X \sqrt{U})^2}{(L/a)^2} + 1 + (L/a)^2 \right] \right] \quad (22d)$$



Choosing the Design Point

Pressure Ratio 5

Fig. 14

the fact that large physical dimensions become necessary to pass a required mass rate of flow.

Moreover the flow coefficient (C_x/U) is restricted at the lower limit by this same inability of a machine to pass sufficient mass flow, and at its upper limit is restricted by the permissible Mach No. incident to the compressor rotor blades. For example this limit may be so set that the relative velocity

$$w_1 \leq .8 \sqrt{KgRT_0}$$

Then since by geometry

$$\frac{w_1}{U} = \frac{(C_x/U)}{\sin \beta_1}$$

there is

$$(C_x/U) \leq .5656 \sin \beta_1 \sqrt{(2g/U^*)KRT_0}$$

Obtaining from (3) for the example in hand

$$(U^2/2g) = 2262/(L/d)_1$$

then, for the compressor inlet:

$$(C_x/U)_1 \leq 1.585 (L/d)_1^{1/2}$$

or

$$\frac{(C_x/U)_1}{(L/d)_1^{1/2}} \leq 1.585$$

Hence, by (25):

$$\delta_{mo} \geq 1.98$$

the fact that large physical dimensions become necessary to pass a required mass rate of flow.

Moreover the flow coefficient $(C_x \sqrt{U})$ is restricted at the lower limit by the same condition of a machine to pass sufficient mass flow, and at its upper limit is restricted by the permissible Mach No. incident to the compressor rotor blades. For example this limit may be set that the relative velocity

$$w_1 \leq 0.8 \sqrt{U_0}$$

Then since by geometry

$$\frac{w_1}{U} = \frac{(C_x \sqrt{U})}{\sin \delta_1}$$

there is

$$(C_x \sqrt{U}) \leq 0.8 \sin \delta_1 \sqrt{U_0}$$

Obtaining from (3) for the example in hand

$$(U \sqrt{U}) = 2000 \sqrt{1/d_1}$$

then, for the compressor inlet:

$$(C_x \sqrt{U}) \leq 1.585 \sqrt{1/d_1} \quad 1/2$$

or

$$\frac{(C_x \sqrt{U})}{1/2} \leq 1.585 \sqrt{1/d_1}$$

Hence, by (25):

$$\delta_{no} \approx 1.98$$

and by a similar procedure for the turbine:

$$\delta_{mt} \approx 1.16$$

Neither of these, it so happens, offers any additional restriction here.

With all the foregoing in mind the dotted rectangles of Fig. 14 were constructed to the (L/d) and (C_x/U) limits, the portion of such rectangles within acceptable limits of then being taken as the area of possible designs. It should be emphasized that the demarcation of these areas is carried out to show that such limits exist, rather than to pretend to lay down their numerical values.

Possible gear ratios for the plant constants chosen are indicated. Direct drive, $G = 1$, appears to be appropriate here since that line is the only one intersecting the rectangle properly. If direct drive is chosen, the gear may be dispensed with and the design latitude remaining is represented by the straight line segment A-B. Re-examination of equations (33) in their latter form above shows a design point near A to be preferred since lower values of aspect ratio assist in reducing turbine and compressor volume. If the point is so chosen that:

$$\delta_{mt} = 2.5$$

$$\delta_{mt} = 1.7$$

then the choice still remains open for the compressor along the arc a-a' and for the turbine along the arc b-b'.

and of a similar procedure for the turbine:

$$\delta_{mt} \approx 1.15$$

Neither of these, it so happens, offers any additional restriction here.

With all the foregoing in mind the dotted rectangle of Fig. 14 was constructed to the (W/B) and (C_X/T) limits, the portion of each rectangle within acceptable limits of then being taken as the area of possible design. It should be emphasized that the determination of these areas is carried out to show that such limits exist, rather than to pretend to lay down their numerical values.

Possible gear ratios for the plant constants chosen

are indicated. Direct drive, $G = 1$, appears to be appropriate here since that line is the only one intersecting the

rectangle properly. If direct drive is chosen, the gear way

be dispensed with and the design latitude remaining is represented by the straight line segment $a-b$. Re-examination of

equation (22) in which latter form shows a design point

near A to be preferred since lower values of speed ratio

result in reducing turbine and compressor volume. If the point

is so chosen that:

$$\delta_{mt} = 2.5$$

$$\delta_{mt} = 1.7$$

then the choice still remains open for the compressor along

the arc $a-a'$ and for the turbine along the arc $b-b'$.

Again (33) shows the lower end of each range to be preferred in order to gain a lower value of (L/d) . Hence the design may be fixed at:

$$(C_x/U)_1 = .5 \qquad (C_x/U)_4 = .88$$

$$(L/d)_1 = .16 \qquad (L/d)_4 = .11$$

and with these in hand the size and weight of the plant may be calculated from the foregoing equations, making use of the air rate, .0105 lb/HP sec., to express the results in terms of power output rather than mass flow rate.

This leads to, for the pressure ratio of five:

$$\begin{aligned} V^* &= V_{cc}^* + V_{tc}^* + V_b^* \\ &= (.49 P^{1/2} + 6.42) 10^{-4} P^* \quad \text{ft.}^3 \end{aligned}$$

and

$$\begin{aligned} W^* &= W_c^* + W_{cc}^* + W_t^* + W_{tc}^* + W_b^* \\ &= (47.22 P^{1/2} + 28.8 P^{1/4} + 17,700/P^{1/4}) 10^{-4} P^* \quad \text{lbs.} \end{aligned}$$

In order to discover the influence of pressure ratio changes on the plant size and weight, the detailed calculations may be repeated for ratios of seven and ten, with other data remaining the same. Following that, by the same criteria as before, there are obtained rectangles which overlies as shown in Fig. 15. When design values of δ , (L/d) and (C_x/U) have been chosen the values for all three pressure ratios may be tabulated as:

Again (33) shows the lower end of each range to be preferred in order to gain a lower value of $(1/d)$. Hence the design may be fixed as:

$$\begin{aligned} (O_x/U)_1 &= 1.8 \\ (1/d)_1 &= 1.11 \end{aligned}$$

and with these in hand the size and weight of the plant may be calculated from the foregoing equations, making use of the air rate, 0.002 lb/ft^3 , to express the results in terms of power output rather than mass flow rate. This leads to, for the pressure ratio of five:

$$\begin{aligned} W &= W_{02} + W_{03} + W_p \\ &= (.49 \text{ lb/ft}^3 + 0.42) 10^{-3} \text{ ft}^3/\text{s} \\ &= 10.2 \end{aligned}$$

and

$$\begin{aligned} W &= W_{02} + W_{03} + W_p \\ &= (.47.22 \text{ lb/ft}^3 + 13.2 \text{ lb/ft}^3 + 17.700 \text{ lb/ft}^3) 10^{-3} \text{ ft}^3/\text{s} \\ &= 10 \end{aligned}$$

In order to discover the influence of pressure ratio changes on the plant size and weight, the detailed calculations may be repeated for ratios of seven and ten, with other data remaining the same. Following that, by the same criteria as before, there are obtained results which overlap as shown in Fig. 12. When design values of δ , $(1/d)$ and (O_x/U) have been chosen the values for all three pressure ratios may be calculated as:

*Effect of Pressure Ratio on
Determination of Aspect Ratio*

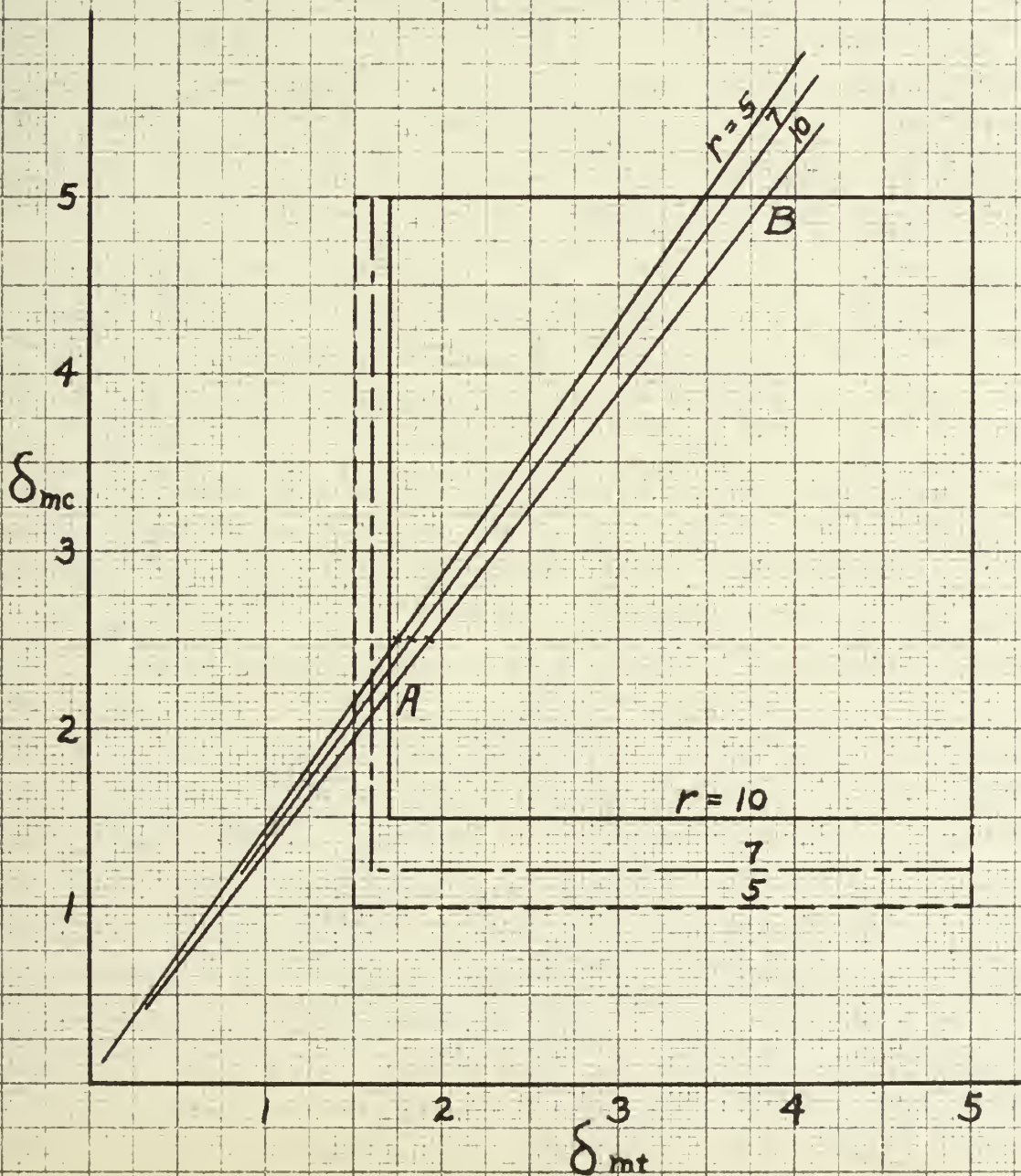
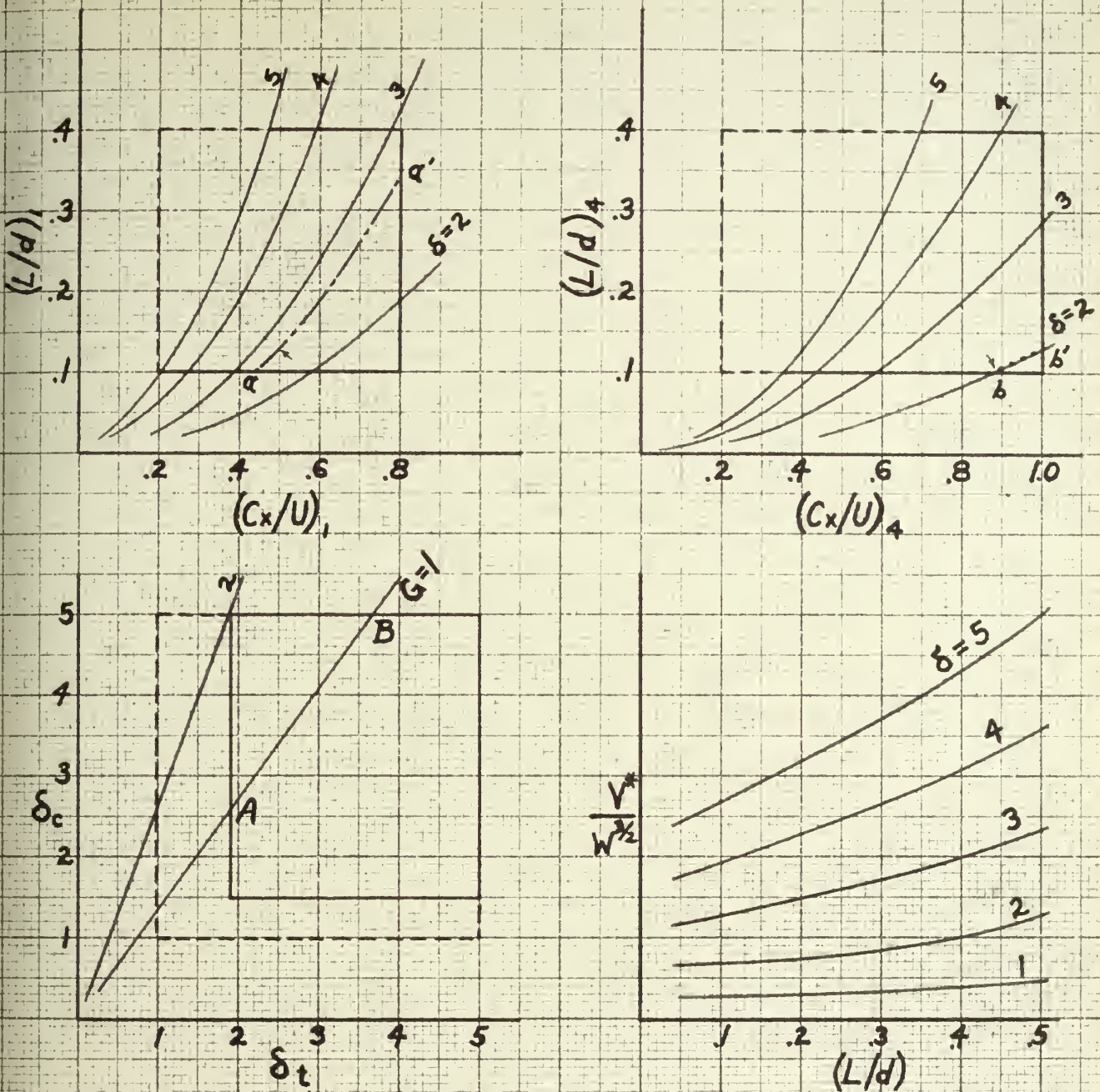


Fig. 15



Choosing the Design Point

Pressure Ratio 7

Fig. 16

r	5	7	10
δ_{mc}	2.5	2.5	2.5
$(C_x/U)_1$.5	.5	.5
$(L/d)_1$.16	.145	.13
δ_{mt}	1.7	1.85	1.95
$(C_x/U)_4$.88	.88	.87
$(L/d)_4$.11	.11	.11

It may be concluded that in the range covered by CBT plants a pressure ratio variation effects little change in the choice of these parameters in turbine and compressor. On the other hand equations (11) et seq. show that the stress parameter Σ , the blade angle and camber β and θ , and the degree of aerodynamic loading under which the blades operate are the three major factors which in the end direct the selections of (L/d) and (C_x/U) . Component stage efficiencies bear on the thermal efficiency and the air rate, and via the latter affect the selection as well.

It may be concluded from the curves (W^*/P^*) of Fig. 17 that, on the basis of minimum plant weight being a desirable factor, it is advantageous at certain pressure ratios to divide a load between two or more identical plants operating in parallel at a net saving in weight. The same possibility with respect to reducing total plant space requirements is suggested by curves (V^*/P) of the same figure. Mitigating against this apparent opportunity to save weight and space,

10	7	8	7
2.5	2.5	2.5	δ_{20}
5	5	5	$(U_x/U)_2$
15	15	15	$(L/U)_2$
1.25	1.25	1.7	δ_{25}
37	38	38	$(U_x/U)_2$
11	11	11	$(L/U)_2$

It may be concluded that in the range covered by GBT plants a pressure ratio variation affects little change in the choice of these parameters in turbines and compressor. On the other hand equations (11) at top show that the stress parameter Σ , the blade angle and number β and ϕ , and the degree of aerodynamic loading which the blades operate are the three major factors which in the end direct the relations of $(L/U)_2$ and $(U_x/U)_2$. Component stage efficiencies depend on the thermal efficiency and the air rate, and via the latter affect the selection as well.

It may be concluded from the curves (W^*/V^*) of Fig. IV that, on the basis of minimum plant weight being a desirable factor, it is advantageous at certain pressure ratios to divide a load between two or more identical plants operating in parallel at a net saving in weight. The same possibility with respect to reducing total plant space requirements is suggested by curves (V^*/V^*) of the same figure. Mitigating against this apparent opportunity to save weight and space,

Effect of Pressure Ratio on Plant Size and Weight

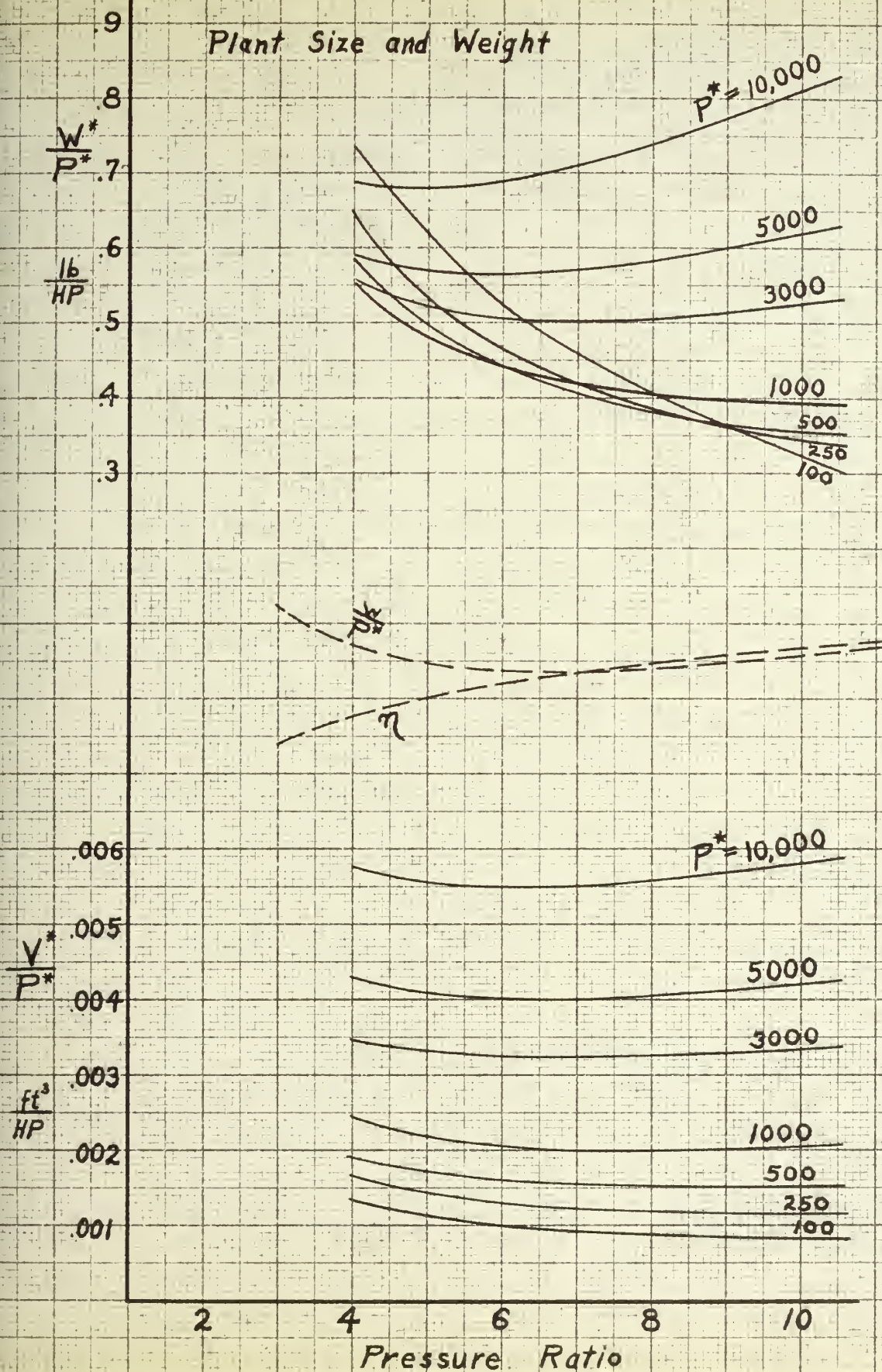
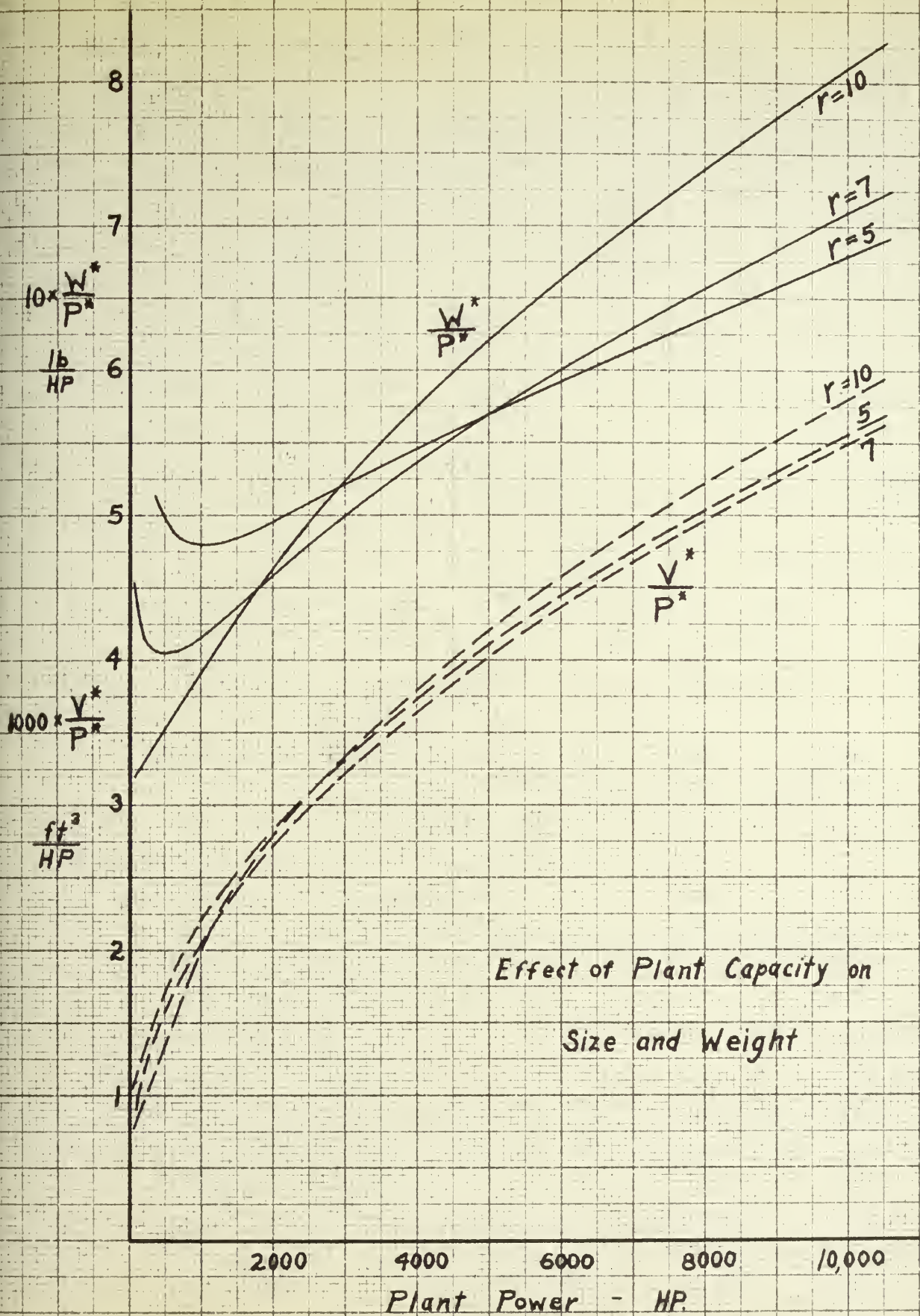


Fig. 17



Effect of Plant Capacity on
Size and Weight

Fig. 18

and in all likelihood actually reversing the trend in an actual installation, is the necessity for duplication of controls, accessories, instrumentation and servicing when sets of smaller unit size are used to fulfill a job requirement.

A quantitative analysis of these two opposing effects is suggested as a suitable subject of further investigation.

and in all likelihood actually reversing the trend in an actual installation, is the necessity for duplication of controls, measurements, instrumentation and servicing when sets of smaller units are used to fulfill a job requirement.

A quantitative analysis of these two opposing effects is suggested as a suitable subject of further investigation.

VIII. APPENDIXSymbolism

A	-	area, annular unless otherwise noted	ft. ²
b	-	blade width, axially	ft.
C	-	absolute velocity	ft./sec.
C _p	-	specific heat at constant pressure	BTU/lb, °F.
D	-	tip diameter, over longest blade	ft.
d	-	pitch diameter, at mid-blade	ft.
F	-	general function; also a general force	
g	-	gravitational constant, 32.2	ft./sec. ²
h	-	enthalpy	BTU/lb
I	-	moment of inertia	ft. ⁴
J	-	mechanical equivalent of heat, 778	ft.lb/BTU
k	-	ratio of specific heats, C _p /C _v	
L	-	blade length, radially	ft.
L*	-	component length	ft.
M	-	Mach No.	
m	-	bending moment	lb.ft
N	-	number of like stages or units	
P	-	pressure	lb/ft. ²
P*	-	power, horsepower unless otherwise shown	
R	-	gas constant, for air 53.3	ft./°F.
R*	-	reheat factor	
r	-	pressure ratio in a component, greater than unity	
S	-	blade spacing, circumferentially, on pitch circle	ft.
σ	-	solidity, b/S	

VIII. APPENDIX

SYMBOLS

1	-	solidity, ϕ	
2	-	circles ϕ , circumferentially, on pitch circle ϕ	
3	-	transverse ratio in a component, greater than unity	
4	-	radius factor	
5	-	gas constant, R in C.G.S.	
6	-	power, horsepower unless otherwise shown	
7	-	pressure	
8	-	number of like stages or units	
9	-	velocity component	
10	-	blade length, radially	
11	-	ratio of specific heats, γ	
12	-	mechanical efficiency of pump, η	
13	-	weight of fluid	
14	-	viscosity	
15	-	gravitational constant, G	
16	-	general function; also a general force	
17	-	blade diameter, at mid-chord	
18	-	tip diameter, open impeller wheel	
19	-	specific heat at constant pressure	
20	-	absolute velocity	
21	-	blade width, axially	
22	-	area, annular unless otherwise noted	

T	-	temperature	°R
t	-	thickness ratio, vs., chord or diameter	
U	-	circumferential velocity on pitch diameter	ft/sec.
V	-	net or working volume	ft. ³
V*	-	component volume, overall	ft. ³
W	-	work	ft.lb/lb.
W*	-	component weight	lb.
w	-	mass flow rate; also relative velocity	lb/sec; ft/sec.
Y	-	cascade clearance factor	
y	-	extreme fiber distance from neutral axis	ft.
Z	-	number of blades in a row	
α	-	angle of absolute velocity with plane of cascade	
β	-	angle of relative velocity with plane of cascade	
Δ	-	a special function of (L/d) and δ	
δ	-	blade aspect ratio, L/b	
ε	-	angle of fluid deflection	
η	-	efficiency, output/input	
θ	-	blade camber angle	
λ	-	the exponent $(k-1)/k \eta_{sc}$	
μ	-	the exponent $(k-1) \eta_{st}/k$	
ν	-	the exponent $(k-1)/k$	
ρ	-	density	lb/ft. ³
Σ	-	stress parameter, $\sigma/\tau\rho_b$	ft.
σ	-	tensile stress	lb/ft. ²
τ	-	taper factor, for stress reduction in rotating blade	
ψ	-	aerodynamic load coefficient	

ψ	-	reciprocating load coefficient
T	-	taper factor, for stress reduction in rotating blade
σ	-	tensile stress
Σ	-	shear stress, σ/τ
ρ	-	density
ν	-	the exponent $(n-1)/n$
μ	-	the exponent $(n-1)/n$
λ	-	the exponent $(n-1)/n$
θ	-	blade camber angle
γ	-	efficiency, output/input
ϵ	-	angle of fluid deflection
δ	-	blade aspect ratio, b/c
Δ	-	a special function of $(1/\delta)$ and δ
β	-	angle of relative velocity with plane of cascade
α	-	angle of absolute velocity with plane of cascade
z	-	number of blades in a row
y	-	extreme fiber distance from neutral axis
γ	-	cascade clearance factor
w	-	mass flow rate; also relative velocity $lb/sec/ft^2$
w_p	-	component weight
w	-	work
v_v	-	component volume, overall
v	-	net or working volume
U	-	differential velocity on pitch diameter
t	-	thickness ratio, w , chord or diameter
τ	-	temperature

ω - angular velocity

rad/sec.

Subscripts

- b - burner or combustor; also blade
- c - compressor; also casing or stator
- g - gas-bending
- j - jet
- m - mean; also pertaining to Mach No.
- n - net
- o - stagnation state
- p - constant pressure
- s - stage; also static
- t - turbine
- x - axial direction

Station identification

- 1 - compressor rotor entrance
- 2 - compressor exit; combustor entrance
- 3 - combustor exit; turbine inlet
- 4 - turbine exit
- 5 - jet discharge

Legend

- 1 - Inlet
- 2 - Compressor exit; turbine inlet
- 3 - Compressor exit; turbine inlet
- 4 - Turbine exit
- 5 - Jet discharge
- 6 - Jet
- 7 - Jet
- 8 - Jet
- 9 - Jet
- 10 - Jet
- 11 - Jet
- 12 - Jet
- 13 - Jet
- 14 - Jet
- 15 - Jet
- 16 - Jet
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- 95 - Jet
- 96 - Jet
- 97 - Jet
- 98 - Jet
- 99 - Jet
- 100 - Jet

Station Identification

- 1 - Compressor inlet
- 2 - Compressor exit; turbine inlet
- 3 - Compressor exit; turbine inlet
- 4 - Turbine exit
- 5 - Jet discharge

Bibliography

- (1) The Spacing of Turbo-Machinery Blading, Especially for Large Angular Deflections. O. Zweifel, Brown-Boveri Review, v.32, p. 436, 1945.
- (2) Lectures on the Development of the Internal Combustion Turbine. Proc. Inst. Mech. Eng., v. 153, 1945.
- (3) Swedish Practice in Compressor Design.
J. R. Schnittger, lectures, M.I.T., 1952.
- (4) Elements of Turbine and Compressor Theory.
W.R. Hawthorne, notes, M.I.T., 1948.
- (5) Effects of Several Design Variables on Turbine Wheel Weight. LaValle and Huppert, NACA TN 1814, 1949.
- (6) Combustion and Combustion Equipment for Aero Gas Turbines. Watson and Clarke, Jour. Inst. of Fuels, v. 21, p. 2, 1947.
- (7) Review of Combustion Phenomena for the Gas Turbine. Shepherd, Trans. ASME, v.73, p.921, 1951.
- (8) Design Features of a 4800-HP Locomotive Gas Turbine Power Plant. Howard, General Electric Tech. Bull. 1457A, 1752, 1753.
- (9) Aerodynamic Turbine with Closed Circuit. Ackeret and Keller, Escher Wyss News, v. 15/16, 1942/43.

References

- (1) The Design of Turbo-Machinery Blades, Especially for Large Axial Compressors. G. Isidori, Brown-Boveri Review, v. 32, p. 422, 1945.
- (2) Lectures on the Development of the Internal Compression Turbine. Trans. Inst. Mech. Eng., v. 153, 1945.
- (3) Swedish Practice in Compressor Design. J. A. Schmittgen, Lectures, N.Y.U., 1952.
- (4) Elements of Turbine and Compressor Theory. V. A. Katchenok, notes, N.Y.U., 1952.
- (5) Effects of Several Design Variables on Turbine Wheel Weight. Isidori and Katchenok, NACA TN 1514, 1949.
- (6) Combustion and Compressor Design for Gas Turbines. Turbines. Isidori and Katchenok, Trans. Inst. Mech. Eng., v. 153, p. 422, 1945.
- (7) Review of Combustion Measurements for the Gas Turbine. Shepherd, Trans. ASME, v. 73, p. 921, 1951.
- (8) Design Features of a 4800-Hp Locomotive Gas Turbine Power Plant. Howard, General Electric Tech. Bull. 1945, 1952.
- (9) Aerodynamic Turbine with Closed Circuit. Anderson and Keller, Mach. Gas News, v. 15/18, 1942/43.



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